PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On the Surface-Tension of Liquids Investigated by the Method of Jet Vibration

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Phil. Trans. R. Soc. Lond. A 1908 **207**, 341-392
doi: 10.1098/rsta.1908.0009

IX. *On the Surface-Tension of Liquids Investigated by the Method of Jet Vibration.**

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Communicated by Lord Rayleigh, *O.M., Pres.R.S.*

Received June 11,—Read June 27, 1907.

[Plates 2–4.]

Contents.

* Abstracted from a response to Det Kongl. Danske Videnskabernes Selskabs (The Royal Danish Scientific Society's) problem in Physics for 1905; delivered October 30, 1906; awarded the Society's gold medal.

Introduction.

Among the large number of methods available for the determination of the surface-tension of liquids that proposed by Lord Rayleigh* stands out with great fundamental advantages. The principle is as follows:—A jet of liquid issuing from a not circular aperture is executing transverse vibrations about its cylindrical configuration of equilibrium. Since the phase of vibration depends upon the time elapsed, it is always the same at the same point in space, and thus the motion is steady and the boundary of the jet a fixed surface showing stationary waves.

Measurements of the corresponding wave-length (λ), the velocity (V), and cross-section (A) of the jet, together with the density (ρ) of the liquid afford the necessary constants for the calculation of the capillary-tension (T) according to Lord Rayleigh's theory of jet-vibration.

The method is free from every supposition respecting the angle of contact. This advantage, however, it has in common with several other methods, especially the following:—

The method of reflection proposed by R. Eötvös† and also used by D. Pekár‡ and G. Zemplén.§

The method of ripples‖ that has been used very much in recent times.

The method of maximum pressure of small air-bubbles proposed by M. Cantor¶ and further developed by R. Feustel.**

Another advantage of Lord Rayleigh's method is that the surface in use is continually renewed. In this manner the capillary-tension can be determined before the surface is $\frac{1}{100}$ second old. This circumstance is of very great importance, as the

* Lord Rayleigh, 'Roy. Soc. Proc.,' 29, p. 71, 1879 ('Papers I.,' p. 377).

† R. Eötvös, 'Wied. Ann.,' 27, p. 448, 1886.

‡ Pekár, 'Zeitschr. f. phys. Chem.,' 39, p. 433, 1902.

§ G. Zemplén, 'Ann. d. Phys.,' 20, p. 783, 1906.

‖ See for instance: L. Matthiesen, 'Pogg. Ann.,' 134, p. 107, 1868; 141, p. 375, 1870; 'Wied. Ann.,' 32, p. 626, 1887; 38, p. 118, 1889. A. Arendt, 'Rep. d. Phys.,' 24, p. 318, 1888. Kelvin, 'Phil. Mag.,' 42, p. 368, 1871; 'Baltimore Lectures,' App. G., London, 1904. Lord Rayleigh, 'Phil. Mag.,' 16, p. 50, 1883 ('Papers II.,' p. 212); 'Phil. Mag.,' 30, p. 386, 1890 ('Papers III.,' p. 383). J. H. Vincent, 'Phil. Mag.,' 43, p. 411, 1897. N. E. Dorsey, 'Phil. Mag.,' 44, pp. 134, 369, 1897. J. A. Craw, in A. Gray, 'A Treatise on Physics,' vol. I., p. 659, London, 1901. L. Grunmach, 'Verh. d. Deutsch. phys. Ges.,' I., p. 13, 1889; 'Ber. d. Akad. d. Wiss.,' Berlin, p. 829, 1900, and p. 914, 1901; 'Ann. d. Phys.,' 3, p. 659, 1900; 4, p. 367, 1901; 6, p. 559, 1901; 7, p. 236, 1902; 9, p. 1261, 1902; 15, p. 401, 1904; 'Festschr.,' L. Boltzmann, p. 460, 1904; 'Wiss. Abh. d. K. Norm.-Aich.-Komm.,' Heft III., p. 101, 1902. Kalähne, 'Ann. d. Phys.,' 7, p. 440, 1902. A. Brümmer, 'Dissertation,' Berlin, 1902. K. Loewenfeld, 'Dissertation,' Berlin, 1905.

¶ M. Cantor, 'Wied. Ann.,' 47, p. 399, 1892; 'Ann. d. Phys.,' 7, p. 698, 1902.

** R. Feustel, 'Ann. d. Phys.,' 16, p. 61, 1905.

main reason of the great discrepancies the different determinations of the surface-tension show in relation to each other is certainly to be found in the variable condition of the tested surface. These irregularities could arise from impurities, for example, fat, oil, or similar substances, even the smallest portion of which is able to produce a great alteration in the surface-tension. Thus Lord RAYLEIGH* has proved that a film of oil not thicker than 2×10^{-6} mm. reduces the surface-tension of water about 28 per cent., and the same author has later† proved that an oil film of even 1×10^{-6} mm. produces a noticeable reduction in the capillary-tension of water. W. C. RÖNTGEN'S‡ experiments show that even an oil film of only $0{\cdot}5 \times 10^{-6}$ mm. is able to appreciably alter the condition of the surface. A. OBERBECK§ has been able to detect the existence of a film of oil that was only $0{\cdot}3 \times 10^{-6}$ mm. thick.

Apart from contamination, the surface can undergo different changes of a chemical and physical nature. In this manner the fluid with which the surface is in contact can cause a chemical change in it. If the liquid under examination is a mixture or a solution the concentration at the surface will in many cases be different from that in the interior.

From the above it is clear that the surface-tension of a liquid is, as a rule, not constant, but varies with the time that has elapsed from the formation of the surface. The value of the capillary-tension, immediately after the formation of the surface, I propose to call the "initial value," while the value of the capillary-tension, when the surface is sufficiently old, is called the "stationary value." Most of the methods for the determination of the surface-tension give values differing from these two limiting values, but it is just these two limiting values that have the greatest interest. Of these the stationary value is of importance in many practical cases, but from a theoretical standpoint the initial value is, without doubt, of the greater interest, as it must stand in a more simple relation to the properties of the liquid than the stationary value, which is dependent upon alien conditions.

With the other methods of measuring, attempts have also been made to work with quite fresh surfaces. GRUNMACH, BRÜMMER and LOEWENFELD have adopted a method of obtaining pure liquid surfaces originally proposed by RÖNTGEN.|| The principle of the method is, that the liquid is conducted from below through the neck of a funnel over the upper horizontal edge of which the liquid flows. This method, however, does not appear to be applicable to all cases (see GRUNMACH, 'Wiss. Abh. d. K. Norm.-Aich.-Komm.,' Heft III., "Experiments with Mercury"), and has at least two faults, firstly, that when the surface renewal takes place somewhat quickly,

* Lord RAYLEIGH, 'Roy. Soc. Proc.,' 48, p. 364 ('Papers III.,' p. 345); 'Phil. Mag.,' 30, p. 386, 1890 ('Papers III.,' p. 383).

† Lord RAYLEIGH, 'Phil. Mag.,' 48, p. 321, 1899 ('Papers IV.,' p. 415).

‡ RÖNTGEN, 'WIED. Ann.,' 41, p. 321, 1890.

§ OBERBECK, 'WIED. Ann.,' 49, p. 366, 1893.

|| RÖNTGEN, 'WIED. Ann.,' 46, p. 152, 1892.

it so easily causes inconvenient currents in the liquid; secondly, the renewal is slowest in the middle of the surface, just at the place which is the subject of the measurement.

FEUSTEL* asserts that the method of maximum pressure of small air-bubbles also gives the tension of a surface that is continually renewed. To this, however, may be replied that the renewal takes place, and must take place, very slowly. If the air-bubbles are produced quickly, the maximum pressure becomes dependent upon the speed with which they are produced.

It will be seen that the surface renewal takes place by Lord RAYLEIGH'S method in a much more effective manner than is possible with the other methods.

Notwithstanding the undoubted fundamental advantages of this method, it has been used in very few cases, for besides Lord RAYLEIGH† it has only been applied by F. PICCARD‡ and G. MEYER.§ Of these PICCARD has made use of the method for the determination of the relative values of the surface-tension of ether, water, alcohol and mercury, but his measurements were carried out with so great an amplitude of vibration (see the plates of his paper, especially Plate VIII., figs. 14, 28 and 29, and Plate X., Photographs 10 and 11) that his results are of very insignificant importance. MEYER has only measured the relative values of the surface-tension of mercury under various conditions.

The explanation of the little use that has been made of this method is to be found in the great difficulties connected with adequate exact determination of the wave-length and cross-section or velocity of the jet. It may be at once remarked here that none of the methods previously used for the determination of these quantities can be taken as satisfactory. It has, therefore, been of the first importance to work out really good methods for the measurements of these quantities.

All the following measurements described here are carried out at ordinary laboratory temperatures.

Even if this method is not so convenient in practice as some of the other methods, that is no great drawback. What is needed in this field of investigation is not any further accumulation of many different measurements, but some more reliable results. Similar reasons have caused the method of ripples, which is just as complicated, to be used a great deal of late.

Theory of the Vibration of a Jet about its Cylindrical Form of Equilibrium.

§1. Before entering into the description of the experimental part of this work it is necessary to set forth a few preliminary remarks on the theory of jet vibrations.

* FEUSTEL, *loc. cit.*

† RAYLEIGH, 'Roy. Soc. Proc.,' 29, p. 71, 1879 ('Papers I.,' p. 377); 'Roy. Soc. Proc.,' 47, p. 281, 1890 ('Papers III.,' p. 341).

‡ F. PICCARD, 'Archives d. Sc. Phys. et Nat.,' (3), 24, p. 561, 1890 (Genève).

§ MEYER, 'WIED. Ann.,' 66, p. 523, 1898.

It will be convenient to set out together the meaning of the symbols employed :—

V = velocity of the jet (cm./sec.).
A = cross-section of the jet (cm.^{2}).
ρ = density of liquid (gm./cm.^{3}).
T = surface-tension (dyne/cm.).
$Q = VA$ = discharge of the jet (cm.^{3}/sec.).
$\lambda_n = 2\pi/k$ = wave-length corresponding to the vibration determined by formula (1) (cm.).

$$I_n(x) = \sum_{s=0}^{s=\infty} \frac{x^{n+2s}}{2^{n+2s} \,.\, \Pi(s) \,.\, \Pi(n+s)}, \qquad I'_n(x) = \frac{dI_n(x)}{dx}.$$

Let us suppose that the polar equation of the surface of the jet is

$$r = a + b_n \cos n\phi \,.\, \cos kz \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

n is an integer greater than 1. The jet is here and in the sequel regarded as horizontal and the plane $\phi = 0$ is also horizontal. According to Lord RAYLEIGH'S* theory the surface-tension is determined by

$$T = \frac{4\sqrt{\pi}}{a^2k^2+n^2-1} \cdot \frac{I_n(ak)}{akI'_n(ak)} \cdot \rho \cdot \frac{V^2A^{3/2}}{\lambda_n^2} = \mu_n(ak) \,.\, \rho \,.\, \frac{Q^2}{A^{1/2} \,.\, \lambda_n^2} \quad . \quad . \quad . \quad (2),$$

where

$$\mu_n(ak) = \frac{4\sqrt{\pi}}{a^2k^2+n^2-1} \cdot \frac{I_n(ak)}{akI'_n(ak)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Vibrations corresponding to different values of n in (1) will be independent of each other.

The development of Lord RAYLEIGH'S theory rests upon certain suppositions, viz. :—

1. That the deviations from the circular-cylinder form are exceedingly small.
2. That the vibrations are executed without any loss of energy.
3. That the original velocity of the jet is the same over the whole cross-section.
4. That the surface-tension is constant.

Each of these hypotheses will now be viewed somewhat closer individually :—

1. This hypothesis is, in practice, impossible to carry out, as it is precisely on the basis of the divergence from a cylindrical form that it is possible to determine the wave-length, and the smaller the divergence the more difficult the determination becomes. To reduce the uncertainty resulting from this, I have investigated a jet of the same liquid partly with large, and partly with proportionally small deviation from

* RAYLEIGH, 'Roy. Soc Proc.,' 29, p. 71, 1879 ('Papers I.,' p. 377).

the cylinder form, and I have similarly used a method for the measurement of the wave-lengths that even permits of a really good determination for small divergences. This matter is more fully considered later.

2. This hypothesis is also of great importance for the development of the theory, but, on the other hand, not satisfactory in practice. The liquid has always some viscosity even though, as in many cases, it is only small. It is, however, possible for the most part to determine the influence of viscosity on the time of vibration, and in this manner to correct the errors caused by it.

The calculation of this correction rests upon the following supposition, which will be very nearly true as long as the viscosity is small :—

> The harmonic vibration of the jet corresponding to the normal co-ordinate b_n is changed by the viscosity to a damped harmonic vibration.

Let the logarithmic decrement of the vibration be δ; we have then

$$N_1 = N \div \sqrt{(1+\delta^2/4\pi^2)},$$

where N_1 is the frequency of vibration with damping, N is the frequency without it.

For the determination of the surface-tension we have instead of (2) the following equation

$$T = \left(1+\frac{\delta^2}{4\pi^2}\right) . \mu_n (ak) . \rho . \frac{Q^2}{A^{1/2} . \lambda_n^2} \quad \ldots \ldots \ldots \quad (4).$$

The experimental determination of δ is described later.

3. The velocity of the thin jets investigated in this work will certainly be nearly the same over the whole cross-section, and correspond to that calculated from the cross-section and the discharge of the jet.

4. The surface-tension is in many cases dependent upon whether the surface extends or contracts (compare, for example, the damping action of oil films on waves); but with the fresh surfaces as used here the surface-tension is certainly very nearly constant.

Calculation of the Coefficients $\mu_n(x)$.

§ 2. The use of the formula [(2), § 1] for the determination of the surface-tension demands the calculation of the coefficients $\mu_n(x)$ determined by the formula [(3), § 1], or

$$\mu_n(x) = \frac{4\sqrt{\pi}}{x^2+n^2-1} \cdot \frac{I_n(x)}{x . I'_n(x)} \quad \ldots \ldots \ldots \quad (1).$$

Here

$$I_n(x) = i^{-n} J_n(ix),$$

where J_n is the BESSEL'S function of the n^{th} order.

Similarly $I'_n(x) = \frac{dI_n(x)}{dx}$. In accordance with the theory of BESSEL'S functions we have

$$I'_n = \frac{n}{x} I_n + I_{n+1}, \quad I'_n = I_{n-1} - \frac{n}{x} I_n, \quad I_{n+1} + \frac{2n}{x} I_n - I_{n-1} = 0 \quad . \quad . \quad . \quad (2).$$

Accordingly we have

$$\mu_n(x) = \frac{4\sqrt{\pi}}{x^2+n^2-1} \cdot \frac{I_n}{xI_{n-1}-nI_n} = \frac{4\sqrt{\pi}}{x^2+n^2-1} \cdot \frac{1}{x \cdot \frac{I_{n-1}}{I_n} - n} \quad . \quad . \quad . \quad . \quad (3).$$

By use of the last formula (2) the values of I_2 and I_3, &c., can be calculated from I_0, I_1, and by substituting these in formula (3) we have $\mu_n(x)$.

In order to facilitate the use of this method for the determination of surface-tension I have calculated a table of the values of $\mu_n(x)$ most commonly used. This table will be found at the end of this paper, and contains the values of $\log_{10} \mu_n(x)$ for $n = 2$, 3, 4, and 6 and for $x = 0{\cdot}00$ to $x = 1{\cdot}00$.

The details of the calculation of this table are given in my original paper.

Calculation of the Vibration of a Jet.

§ 3. In accordance with Lord RAYLEIGH'S theory the vibration of a jet can now be determined when the velocity and original cross-section is known, although, as previously emphasised, the theory is only available for small deviations from the circular form.

The circumference at the original cross-section is determined by

$$r = a_0 + F(\phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

By help of FOURIER'S series this equation can be written as

$$r = a_0 + \sum_{n=2}^{n=\infty} b_n \cdot \cos(n\phi + \epsilon_n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

when, if necessary, the value of a_0 is changed so that b_0 vanishes, and the origin of co-ordinates is changed so that b_1 becomes zero.

Each term in (2) can be taken alone and the resulting vibration of the jet can be calculated as the sum of all the vibrations corresponding to the different values of n in (2).

Thus it is only necessary to consider

$$r = a_0 + b_n \cdot \cos(n\phi + \epsilon_n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

The wave-length λ_n corresponding to the vibration (3) is, according to [(2), § 1],

$$\lambda_n = C \cdot \sqrt{\mu_n(a_0 k)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

where

$$C = \rho^{1/2} \cdot A^{3/4} \cdot V \cdot T^{-1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

is independent of n.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The determination of λ_n is easiest carried out in the following manner:—

$$\lambda'_n = C\sqrt{\mu_n(0)}\,; \quad \lambda''_n = C\,.\sqrt{\mu_n\left(a_0\frac{2\pi}{\lambda'}\right)}\,; \quad \lambda'''_n = C\,.\sqrt{\mu_n\left(a_0\frac{2\pi}{\lambda''_n}\right)},\ \&c.$$

In nearly all cases it will be sufficiently correct to take $\lambda_n = \lambda''_n$.

If there is only one vibration (3) present, the equation for the surface of the jet will accordingly be

$$r = a_0 + b_n\cos(n\phi+\epsilon_n)\,.\cos(2\pi z/\lambda_n).$$

If all the partial vibrations are taken in the same manner, we have, by the addition of the results, the following equation for the surface of the jet

$$r = a_0 + \sum_{n=2}^{n=\infty}.\,b_n\,.\cos(n\phi+\epsilon_n)\,.\cos(2\pi z/\lambda_n) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

As an example, we will here calculate the vibration of a jet the original cross-section of which is an ellipse with the axis $2a$ and $2b = 2(a-\delta)$.

The polar equation of the ellipse is

$$r = \frac{ab}{\sqrt{a^2\sin^2\phi + b^2\cos^2\phi}} = \frac{a-\delta}{\sqrt{1-x\cos^2\phi}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

where

$$x = 2\frac{\delta}{a} - \frac{\delta^2}{a^2}.$$

By expanding in series

$$\begin{aligned} r = (a-\delta)\Big\{&\tfrac{1}{4}\left(1+\frac{1}{\sqrt{1-x}}\right)+\tfrac{1}{2}\,.\frac{1}{\sqrt{1-\frac{1}{2}x}} \\ &+\cos 2\phi\,.\left(\tfrac{1}{2}\cdot\tfrac{1}{2}x+\tfrac{1}{2}\cdot\tfrac{1.3}{2.4}x^2+\tfrac{15}{32}\cdot\tfrac{1.3.5}{2.4.6}\cdot x^3+\tfrac{7}{16}\cdot\tfrac{1.3.5.7}{2.4.6.8}x^4+\tfrac{105}{256}\cdot\tfrac{1.3.5.7.9}{2.4.6.8.10}x^5+\ldots\right) \\ &+\cos 4\phi\,.\left(\tfrac{1}{8}\cdot\tfrac{1.3}{2.4}x^2+\tfrac{3}{16}\cdot\tfrac{1.3.5}{2.4.6}x^3+\tfrac{7}{32}\cdot\tfrac{1.3.5.7}{2.4.6.8}x^4+\tfrac{15}{64}\cdot\tfrac{1.3.5.7.9}{2.4.6.8.10}x^5+\ldots\right) \\ &+\cos 6\phi\,.\left(\tfrac{1}{32}\cdot\tfrac{1.3.5}{2.4.6}x^3+\tfrac{1}{16}\cdot\tfrac{1.3.5.7}{2.4.6.8}x^4+\tfrac{45}{512}\cdot\tfrac{1.3.5.7.9}{2.4.6.8.10}x^5+\tfrac{55}{512}\cdot\tfrac{1.3.5.7.9.11}{2.4.6.8.10.12}x^6+\ldots\right) \\ &+\cos 8\phi\,.\left(\tfrac{1}{128}\cdot\tfrac{1.3.5.7}{2.4.6.8}x^4+\tfrac{5}{256}\cdot\tfrac{1.3.5.7.9}{2.4.6.8.10}x^5+\tfrac{33}{1024}\cdot\tfrac{1.3.5.7.9.11}{2.4.6.8.10.12}x^6+\ldots\right) \\ &+\ldots\Big\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8). \end{aligned}$$

By inserting $x = 0{\cdot}4$ or $\delta/a = 0{\cdot}2254$ in formula (8) this formula is reduced to

$$\begin{aligned} r = 0{\cdot}7746a\,(&1{\cdot}1318 + 0{\cdot}1440\cos 2\phi + 0{\cdot}0138\cos 4\phi \\ &+ 0{\cdot}0015\cos 6\phi + 0{\cdot}0002\cos 8\phi + \ldots) \quad . \quad . \quad (9). \end{aligned}$$

The further calculations are based upon the following constants: V = 240 cm./sec., A = $0{\cdot}066$ cm^2., $a = 0{\cdot}1647$ cm., $\rho = 1$, and T = 73 dyne/cm. The wave-lengths corresponding to n = 2, 4, and 6, calculated by the above given means, are

$$\lambda_2 = 3{\cdot}932\text{ cm.}\,; \quad \lambda_4 = 1{\cdot}227\text{ cm.}\,; \quad \lambda_6 = 0{\cdot}646\text{ cm.}$$

The equation of the jet becomes then

$$r = 0{\cdot}7746 \,.\, 0{\cdot}1647 \left(1{\cdot}1318 + 0{\cdot}1440 \cos 2\phi \,.\, \cos \frac{2\pi z}{3{\cdot}932} + 0{\cdot}0138 \cos 4\phi \,.\, \cos \frac{2\pi z}{1{\cdot}227} + \ldots\right)$$

$$= 0{\cdot}1444 + 0{\cdot}01837 \cos 2\phi \,.\, \cos \frac{2\pi z}{3{\cdot}932} + 0{\cdot}001760 \,.\, \cos 4\phi \,.\, \cos \frac{2\pi z}{1{\cdot}227}$$

$$+ 0{\cdot}0001913 \cos 6\phi \,.\, \cos \frac{2\pi z}{0{\cdot}646} + \ldots \quad . \quad (10),$$

from which the co-ordinates for every point of the surface of the jet can be calculated. The profile line resulting from $\phi = 0$ in (10) has special interest. This line is shown in fig. 1. In order better to judge the form of the curves the height is enlarged

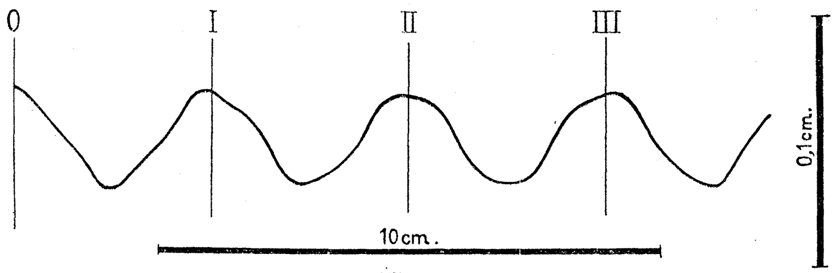


Fig. 1.

fifty times in relation to the length. It can be seen that the form of the curve is mainly determined by the original vibration corresponding to $n = 2$, but that at the same time also the other vibrations cause perceptible deviations.

In the measurements made by Lord Rayleigh, Piccard, and Meyer the wave-length λ_2 is determined as the length between two successive summits of the profile line of the jet. It can be seen in fig. 1, that this length can vary and deviate somewhat from the wave-length λ. In order to illustrate the size of these deviations for the jet corresponding to (10), drawings of the summits of the profile line are shown in fig. 2. By calculation it is found that r is maximum for $z = 0$, $z = 3{\cdot}932 - 0{\cdot}109$ cm., $z = 2 \,.\, 3{\cdot}932 - 0{\cdot}171$ cm., and $z = 3 \,.\, 3{\cdot}932 + 0{\cdot}211$ cm.

As above stated, $\lambda_2 = 3{\cdot}932$ cm.

The wave-lengths measured in this manner are

$$\lambda^{0\text{-}I} = 3{\cdot}823 \text{ cm.}; \quad \lambda^{I\text{-}II} = 3{\cdot}870 \text{ cm.}; \quad \lambda^{II\text{-}III} = 4{\cdot}314 \text{ cm.}$$

The errors stated in per cent. of λ_2 are respectively $-2{\cdot}6$, $-1{\cdot}6$, and $+9{\cdot}7$.

The error can be reduced by taking the mean value of several lengths. On the other hand, the amplitudes of the supplementary vibrations have been greater in proportion to the fundamental vibration with almost all the measurements up to now than in the instance mentioned here.

Even with relative measurements as those made by Meyer* these reasons can

* Meyer, *loc. cit.*

have weight, as the wave-lengths λ_2, λ_3, and λ_4 are dependent upon the surface-tension, and a slight variation, for example in λ_2/λ_4, can to some extent alter the wave-lengths measured.

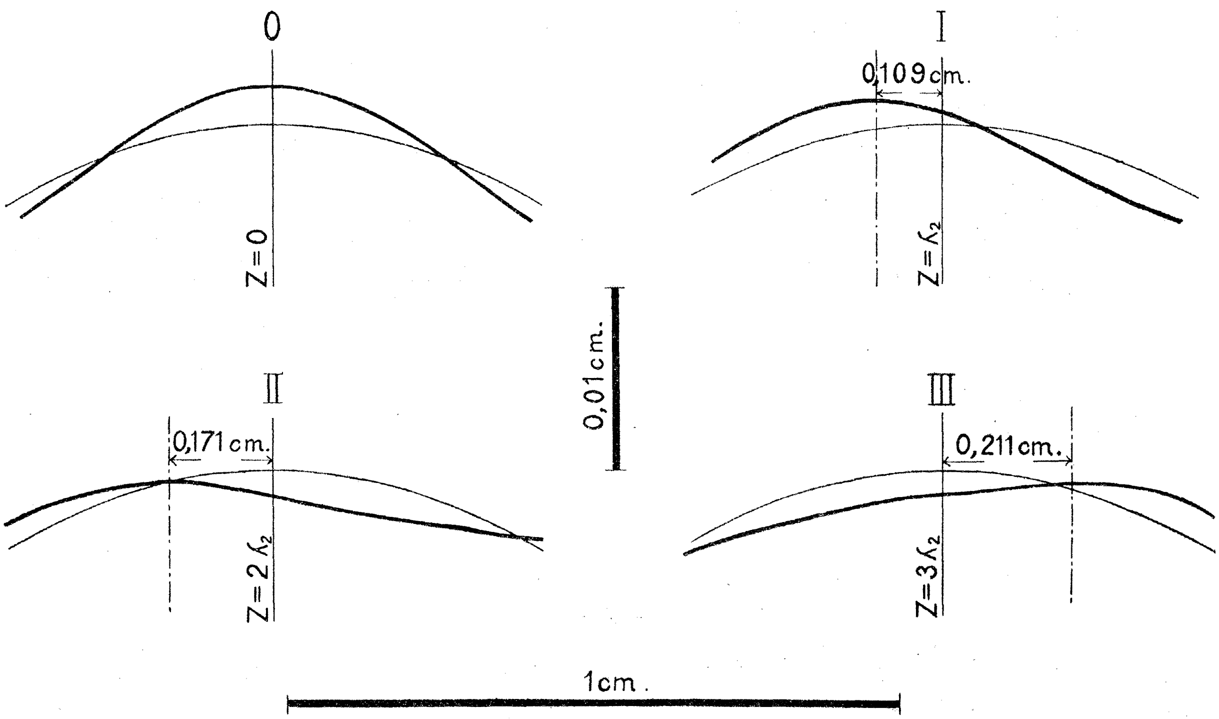


Fig. 2.

Preliminary Investigations.

Arrangement for Keeping the Pressure Constant.

§ 4. Before I begin the description of the different measurements I will mention a method for the production of a constant pressure by use of ordinary tap-water, as I have used this means in almost all the preliminary investigations.

It has hitherto always been an inconvenience in experiments with jets that the pressure varies continuously as the fluid runs out. To avoid this variation it is not advisable to use the simple method of renewing the quantity of discharge by a corresponding inflow of fluid, as this arrangement produces disturbances in the fluid mass, causing irregularities in the jet. All those who have worked with jets know how great is the demand for rest in the reservoir, and how exceedingly sensitive the jets are to external influence.

Lord Rayleigh* states: "The jet is exceedingly sensitive to disturbances in the reservoir, and no arrangement hitherto tried for maintaining the level of the water has been successful."

* Lord Rayleigh, 'Roy. Soc. Proc.,' 29, p. 71, 1879. ('Papers I.,' p. 380.)

After a number of experiments I have come to the conclusion that the following arrangement for this special use is perfectly satisfactory :—

From the tap the water is conducted direct through a rubber tube to the spout of the funnel T (fig. 3), which is fastened with sealing-wax to the neck of the bottle F.

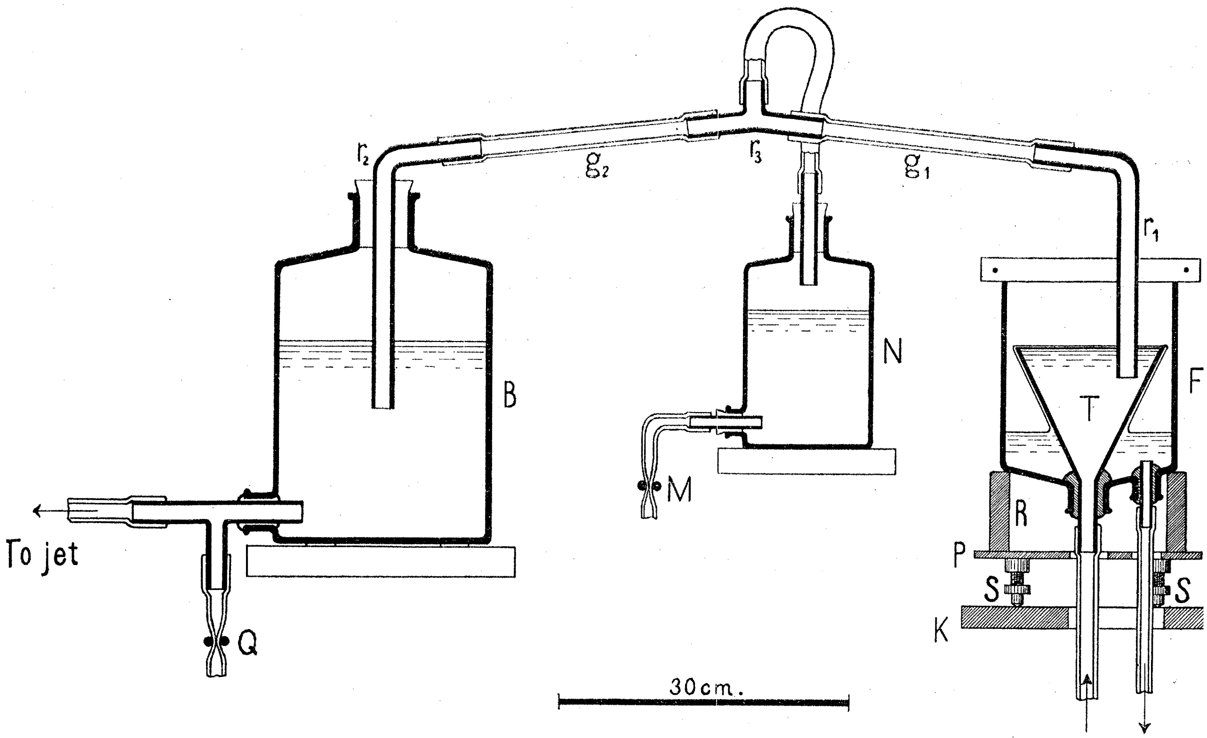


Fig. 3.

This rests through a wooden frame R on a metal plate P, which is provided with three adjustable screws S. The whole is borne on a bracket K, placed on an outer wall. The bottle F is open above and is provided with an outlet from below as shown in the figure. By help of the adjustable screws the upper edge of T is kept horizontal.

The water coming from the supply pipe will run over the edge of the funnel in the form of a thin layer, and the height of the surface of water in the funnel will only be very slightly dependent upon the speed of the supply, so that the unavoidable variations in the pressure of the supply pipe will practically have no influence.

The water in the funnel T is in connection with the water in the reservoir B through a syphon made of glass tubes r_1, r_2, r_3 and rubber tubes g_1 and g_2; the surface in B will keep the same height as the surface of water in T. B is provided below with a tubulure that serves for the introduction of one branch of a T-tube, the other two branches of which are provided with rubber tubes, the one serving the jet apparatus, whilst the other is only used for filling or emptying the reservoir.

It follows from the above that when the quantity of water supplied to T from the supply pipe is greater than that used in the jet apparatus, the surface of water in B will keep itself practically constant, independently of the quantity used in the jet apparatus. This is, of course, on the understanding that the diameter of the syphon is sufficiently large.

Experiments have shown that with this arrangement, and for jets not exceeding 2 mm. in diameter, the water surface in B will vary at most 0·2 mm., and the apparatus can stand and operate by itself day and night.

The above-mentioned arrangement is especially convenient for investigation of a jet produced from ordinary tap-water, and was used in practically all the preliminary investigations for judging the exactness and practicability of the methods of measuring. As these investigations take, as a rule, a long time, it is very important that the pressure be kept constant.

For other fluids that are available only in limited quantities this method cannot, of course, be used. In these cases the author has, as a rule, employed the usual *modus operandi* with decreasing pressure. This will be more fully explained under the description of the experiments.

Determination of the Cross-section of the Jet.

§ 5. When using the present method for determination of the surface-tension it is necessary besides the wave-length to know two of the three following quantities: velocity of the jet, the sectional area of the jet, and the discharge. This last named is easiest to determine with sufficient exactness, and will, therefore, in every case be measured. The choice then remains between measuring the velocity of the jet or the cross-sectional area, but before making this choice I will give a short summary of the methods that are available at this moment to determine the velocity and sectional area of a jet.

The velocity V can be determined by use of TORRICELLI'S formula $V = \alpha\sqrt{(2gH)}$, where H is the pressure, g the acceleration of gravity and α a coefficient. Many experiments have been made to investigate the exactness of TORRICELLI'S formula. The results of these investigations are, mainly: for fluids with little viscosity, with not too high pressure, and, lastly, with holes the diameter of which exceeds 5 mm., the formula is practically correct, as the coefficient α is very nearly equal to 1; for water $\alpha = 0{\cdot}97$ to $0{\cdot}99$. Special reference can be made here to TH. VAUTIER'S* careful investigations on this subject.

With this in view, it was the author's original intention to determine the velocity of the jet in this manner; it, however, soon appeared that, just in the circumstances which have especial interest in the present instance, the deviations from TORRICELLI'S formula are very important. It is for this reason that, so as not to use too great a quantity of fluid, it is necessary to use thin jets, for example with a diameter of

* TH. VAUTIER, 'Compt. Rend.,' 103, p. 372, 1886; 'Thèse prés. à la Fac. de Science d. Paris,' 1888; 'Ann. Chim. Phys.,' (6), 15, p. 433, 1888; 'Journ. d. Phys.,' (2), 8, pp. 301, 396, 1889.

1 millimetre. Several influences result from this which with a greater diameter of jet have only secondary importance, but here take a prominent part. I shall quite superficially treat these influences here, as I hope later to have an opportunity to give a more exhaustive account of this subject.

In TORRICELLI's formula H indicates the pressure measured from the jet to the fluid's surface; but this in reality must be reduced by the pressure produced by the surface-tension in the interior of the jet. Let d cm. be the diameter of the jet, ρ the density, and T dyne/cm. the surface-tension, then will the pressure produced by the surface-tension correspond to a head of liquid, the height of which, h cm., is determined by

$$h = \frac{2T}{dg\rho} \quad \ldots \ldots \ldots \ldots (1).$$

If we take, for example, for water $d = 0{\cdot}1$ cm., $\rho = 1$, $T = 73{\cdot}5$ dyne/cm., we have $h =$ circa 1·5 cm. As can be seen, it is a correction which is not quite infinitesimal, though but little attention has been given to it. C. CHRISTIANSEN* is probably the first who has commenced the investigation with special regard to the conditions under discussion. Some experiments of M. ISARN† also confirm the above view. He determined the time that elapsed for 141 cm³. of fluid to run through a circular hole of a diameter of 0·8 mm. with the pressure varying from 11·8 cm. to 9·0 cm. and found

for water 290 seconds,
,, alcohol 270 ,,

These two measurements will now be calculated with reference to the correction mentioned before (1), it being supposed that the diameter of the jet in both instances was $d = 0{\cdot}07$ cm.

For water we take $\rho = 1$, $T = 73{\cdot}5$ dyne/cm.; for alcohol we take $\rho = 0{\cdot}8$, $T = 22{\cdot}0$ dyne/cm.

In accordance with this the above correction will be for water $h = 2{\cdot}14$ cm., for alcohol $h = 0{\cdot}80$ cm.

The effective pressure will according to this be

$$\text{For water} \quad H = \tfrac{1}{4}\left(\sqrt{11{\cdot}8 - 2{\cdot}14} + \sqrt{9{\cdot}0 - 2{\cdot}14}\right)^2 = 8{\cdot}20 \text{ cm.}$$

$$\text{,, alcohol} \quad H = \tfrac{1}{4}\left(\sqrt{11{\cdot}8 - 0{\cdot}80} + \sqrt{9{\cdot}0 - 0{\cdot}80}\right)^2 = 9{\cdot}53 \text{ cm.}$$

The total discharge will be

$$\text{For water} \quad \pi \,.\, \frac{0{\cdot}07^2}{4} \cdot \sqrt{2{\cdot}981 \,.\, 8{\cdot}2} \,.\, 290 = 141{\cdot}3 \text{ cm}^3.$$

$$\text{,, alcohol} \quad \pi \,.\, \frac{0{\cdot}07^2}{4} \cdot \sqrt{2{\cdot}981 \,.\, 9{\cdot}53} \,.\, 270 = 141{\cdot}8 \text{ cm}^3.$$

* C. CHRISTIANSEN, 'Overs. Vidensk. Selsk. Forh.,' p. 65, 1901; 'Ann. d. Phys.,' 5, p. 436, 1901.

† M. ISARN, 'Journ. d. Phys.' (1), 4, p. 167, 1875.

Thus the difference shown in the time of outflow is wholly explained in this manner.

ISARN himself explains the difference mentioned as originating from different contractions, and calculates on the basis of such measurements the coefficient of contraction. There can be no doubt, however, that this is incorrect, as the influence of the surface-tension on the coefficient of contraction certainly is not great, as will be shown later.

If this reduction of the velocity on account of the capillary-pressure in the jet were the only deviation from TORRICELLI's formula, it could be corrected and the velocity accordingly calculated; but as the diameter of the jet becomes smaller the value of α is also reduced and this coefficient becomes to a great degree dependent upon the nature of the edge of the hole, so that in every case it is necessary to determine the value of α, or, in other words, determine the velocity of the jet in another manner.

Lord RAYLEIGH*, who used this method to determine the velocity of the jet, says with regard to it: "The pressure at any moment of the outflow could be measured by a water manometer read with a scale of millimeters. Some little uncertainty necessarily attended the determination of the zero point; it was usually taken to be the reading of the scale at which the jet ceased to clear itself from the plate on the running out of the water."

According to the above, this method can not be taken as a satisfactory solution of the question.

Direct measurement of the velocity of the jet can be made in several manners. TH. VAUTIER† added small drops of another fluid and determined the velocity of the drops by taking photographs on a plate moving with a known speed. The method seems to be good so long as the diameter of the jet is not too small (in VAUTIER's experiments the diameter of the hole was 5·76 mm.), but with small diameters the method is useless on account of the risk of noticeable change both in the surface-tension and in the coefficient α on account of the additional alien liquid.

Another and simpler method‡ is to determine the velocity by help of the geometric form of the jet. This method can also give satisfactory results for thick jets, but for thin ones it is of no value.

Besides the previously mentioned reduction of velocity on account of the capillary pressure in the jet, the surface-tension produces other differences in the velocity. The presence of the jet is inseparably connected with a continual production of new fluid surface, and the requisite energy is essentially taken from the kinetic energy of the jet as its horizontal velocity reduces. The loss of pressure, h_1, corresponding to this reduction is easily found to be

$$h_1 = \frac{4T}{d\rho g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

* Lord RAYLEIGH, 'Roy. Soc. Proc.,' 29, p. 71, 1879 ('Papers I.,' p. 375).

† TH. VAUTIER, *loc. cit.*

‡ See WINKELMANN, 'Handbuch d. Phys.,' I., "Ausfluss und Strahlbildung," F. AUERBACH, 1891.

This reduction is thus seen to be double that originating from the capillary-pressure for a water jet with a diameter of 1 mm. becomes h_1 = circa 3·0 cm.

This fact has also been the subject of but little attention, though A. DUPRÉ* has undertaken some interesting experiments about the height to which a jet can rise.

A closer examination of how the reduction of velocity corresponding to h_1 spreads itself over the jet I am obliged to leave to another opportunity, only a particular characteristic fact being named here. If the pressure is reduced more and more so that it comes near to the value h_1+h, the jet can be observed to deviate more and more from a parabola, the curvature of the jet just outside the hole becoming much too great. If the pressure is reduced almost to h_1+h, the jet first runs a short distance horizontally and then falls vertically down. If we determine the discharge and cross-section of the jet, the velocity in the horizontal part of the jet can be calculated, and it will be observed that this velocity is almost equal to that corresponding to the pressure h_1. It is very difficult to maintain the jet in the above position; the slightest disturbance will cause the jet to cease.

It is also possible to determine the velocity of a jet by measuring the pressure it produces by normal impact on a sufficiently large plane surface. Measurements in this manner have been made, for example, by BUFF,† who worked with a jet with diameter from 5 to 7 mm. This method seems to give quite reliable results and may most probably be available even for much smaller jet thicknesses, but the difficulties connected with its use will be, in consequence, considerably greater. The balance used must, in such a case, be made very sensitive, and it will probably then be difficult to keep the sensitiveness constant. In the use of this method there must also be taken into consideration a correction resulting from the surface-tension. The pressure measured must be reduced by the capillary-pressure in the jet multiplied by the area of its cross-section, in other words, by

$$\frac{2\mathrm{T}}{d} \cdot \frac{\pi}{4} \cdot d^2 = \frac{\pi}{2} \mathrm{T} d.$$

At the same time the surface-tension along the jet's circumference, or $\mathrm{T}\pi d$, must be added to the pressure measured. The final correction will accordingly be $+\frac{1}{2}\pi \mathrm{T} d$.

The cross-section of a jet has hitherto, as a rule, been determined by direct geometrical measurements, which ordinarily take place in such a manner that the points of some micrometer screws are brought exactly to touch the surface of the jet. In this manner a sufficient exactness can be reached, as a rule, with thick jets. The condition is quite the reverse when the diameter of the jet is only about 1 millimetre. In this case it will most probably be impossible to get even a moderately satisfactory exactness. In this respect a great progress has been made by the elegant method

* A. DUPRÉ, 'Théorie mécanique de la chaleur,' p. 376, Paris, 1869.

† BUFF, 'POGG. Ann.,' 137, p. 497, 1869.

proposed by K. PRYTZ* and based on the optical contact between a microscope and a reflecting surface. It is possible in this manner to determine the diameter of a circular

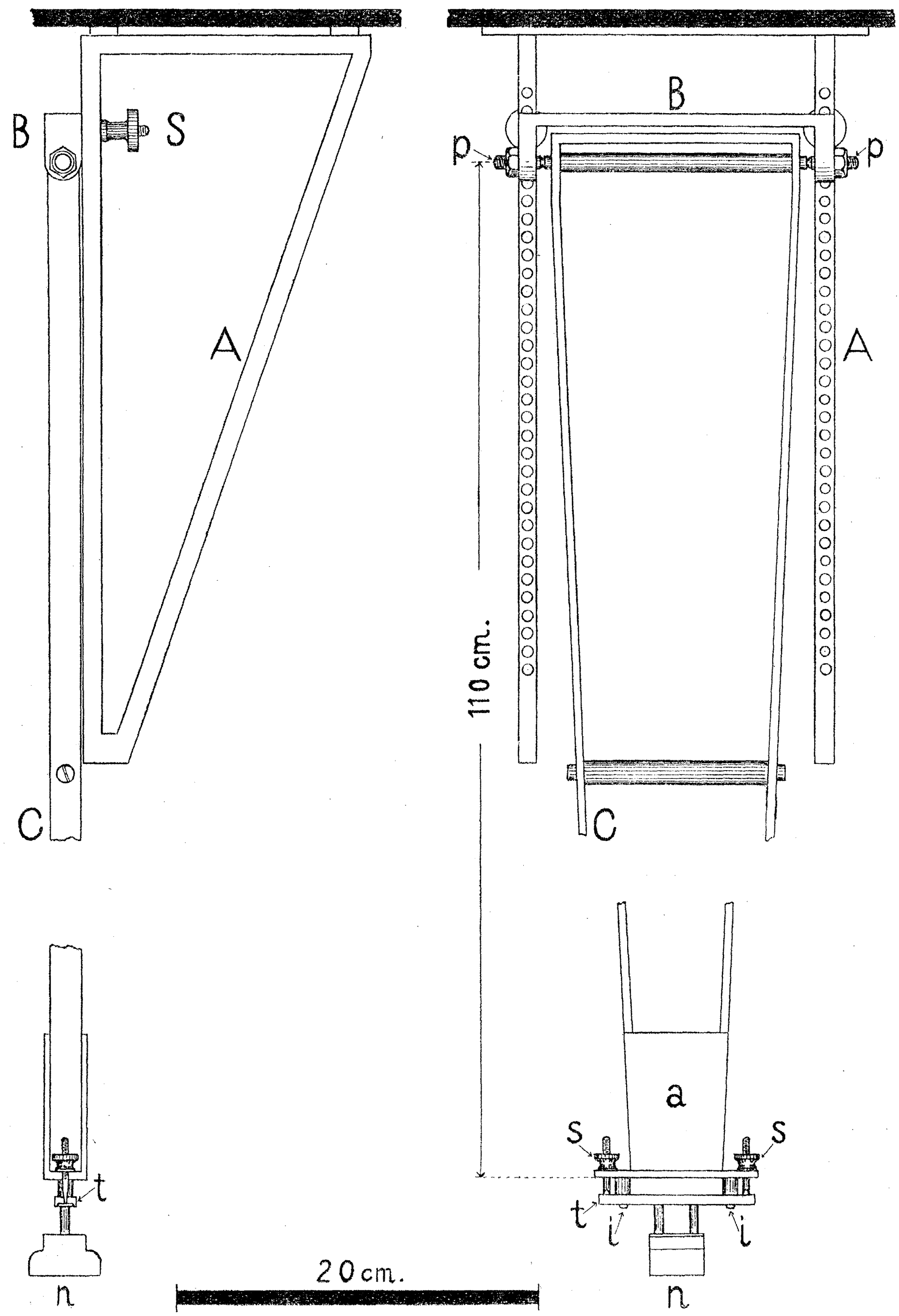


Fig. 4.

jet with great exactness, but with not circular jets the method, unfortunately, becomes impracticable.

* PRYTZ, 'Overs. Vidensk. Selsk. Forh.,' p. 17, 1905; 'Ann. d. Phys.,' 16, p. 733, 1905.

§ 6. As none of the known methods for the determination of the velocity and cross-section of thin jets are quite satisfactory, I have worked out a new method for the determination of the cross-sectional area of a jet.

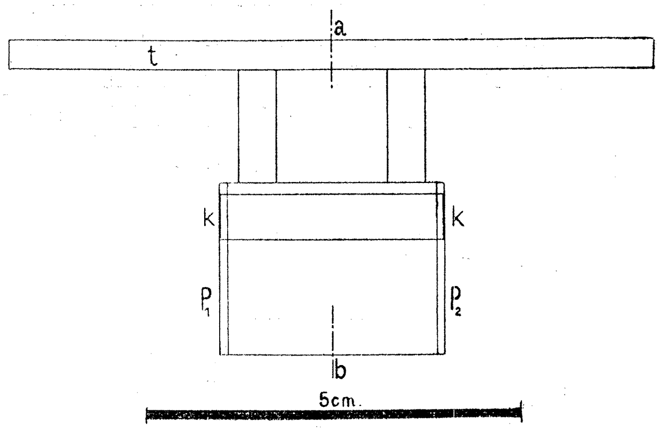


Fig. 5.

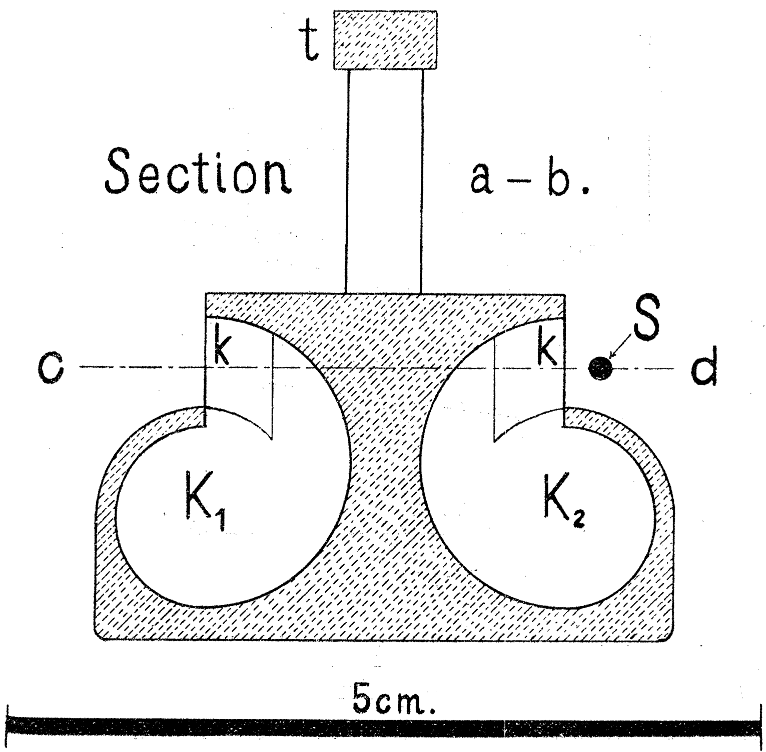


Fig. 6.

The principle of it is the following: A definite length of a jet is taken and weighed, and on the basis of the weight and the density of the liquid the sectional area of the jet is calculated.

The taking of this definite length of the jet is accomplished by help of a "jet-catcher," shown in figs. 4–7. It consists of two cylindrical vessels K_1 and K_2, the

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

ends of which are the steel plates P_1 and P_2. The edges of these steel plates, *k*, are knife-formed and effect the cutting of the jet, as described later.

The "jet-catcher" itself is, as is shown in figs. 5–6, arranged on a pendulum C, turning about the axis *p–p*. In order to vary the height of the pendulum the stands A are furnished with a number of holes for the screws S, as shown in the figure. The "jet-catcher" is fastened to the pendulum through the cross-piece *t* by means of the screws *s* (see fig. 4). The exact position is secured by means of two cones *i*, which fit in the corresponding holes in the cross-piece *t*.

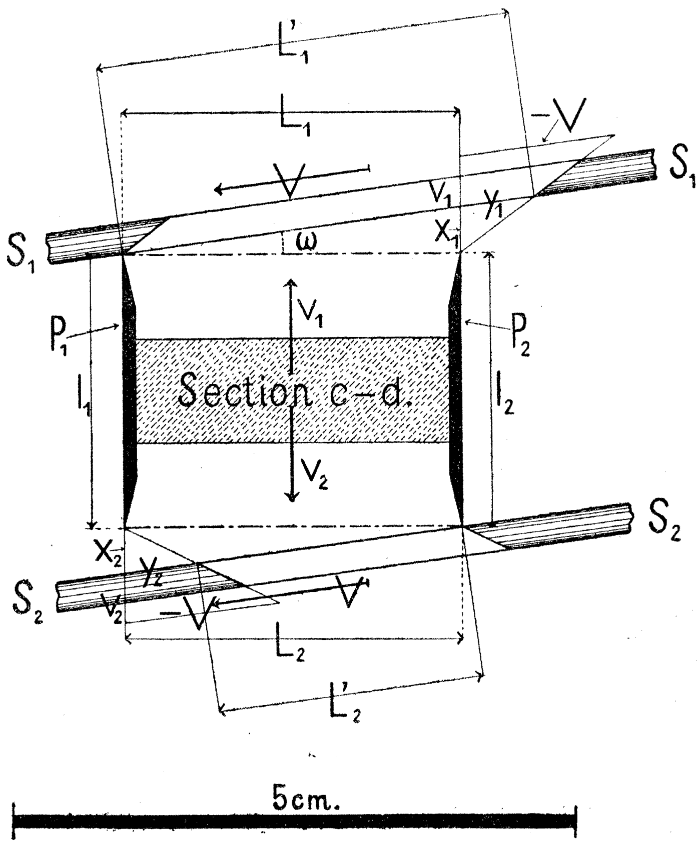


Fig. 7.

The axis of the pendulum is arranged perpendicularly over, and parallel to, the horizontal jet at such a height that the position of the jet in relation to the "jet-catcher" is about as shown in fig. 6, where S indicates the jet.

If the pendulum is caused to swing, the "jet-catcher," every time it passes the jet, will cut out a portion of it. If the jet is perpendicular to the edges *k*, and parallel to the plane through corresponding edges, the length of the piece that is cut out of the jet each time is equal to the distance between the edges *k*. With a complete swing (both forwards and backwards) the total length L of the portions of the jet cut off (see fig. 7) is

$$L = L_1 + L_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

If the jet is parallel to the plane through the edges, but makes an angle of $90°-\phi$ with these, then the total length of the portions cut off during a complete swing will become

$$L = \frac{1}{\cos\phi}(L_1+L_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

If the jet is at right angles to the edges, but makes the angle ω with the plane through the edges (see fig. 7), the conditions are a little more complicated. In the figure the jet is drawn in two positions, S_1 and S_2. The one position, S_1, gives a picture of what takes place with the "jet-catcher" moving in one direction, S_2 gives a corresponding picture of the movement in the opposite direction. V is the velocity of the jet. With regard to the other symbols, reference is made to the figure.

We have

$$L'_1 = L_1 \sec\omega + y_1, \quad y_1 = V/v_1 \, . \, x_1, \quad x_1 = L_1 \tan\omega,$$

also

$$L'_1 = L_1 (\sec\omega + V/v_1 \, . \tan\omega) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

In the same manner we get

$$L'_2 = L_2 (\sec\omega - V/v_2 \, . \tan\omega) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

By the addition of (3) and (4) we get

$$L = L'_1 + L'_2 = (L_1+L_2)\left[\sec\omega + \tfrac{1}{2} V \tan\omega \,(1/v_1 - 1/v_2)\right] . \quad . \quad . \quad . \quad (5);$$

as the last term is so small that we can take $L_1 = L_2$ without any appreciable error.

If $v_1 = v_2$,

$$L = (L_1+L_2) \, . \sec\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

In practice v_1 and v_2 will have almost the same value, although the velocity will naturally be somewhat smaller each time the "jet-catcher" passes the jet. To investigate the influence of this difference in velocity we take $v_2 = 0{\cdot}9 v_1$, $V = v_1$.

The equation (5) then becomes

$$L = (L_1+L_2) \, . (\sec\omega - 0{\cdot}0555 \tan\omega) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

By $v_1 = 0{\cdot}9 v_2$ and $V = v_2$, equation (5) becomes

$$L = (L_1+L_2) \, . (\sec\omega + 0{\cdot}0555 \tan\omega) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

To judge the influence of the angle ω the Table I. is available, which also contains the corresponding values for $v_2 = 0{\cdot}95 v_1$ and $v_1 = 0{\cdot}95 v_2$.

In practice the ratio between v_1 and v_2 will still more approach to 1. As can be seen, no especially great exactness is required in the adjustment of the "jet-catcher" relatively to the jet.

TABLE I.

	$\omega =$	1°.	2°.	3°.	4°.	5°.
$v_2 = 0 \cdot 90 v_1$	$\sec\omega - 0 \cdot 055 \tan\omega$	0·99918	0·99867	0·99846	0·99856	0·99896
$v_1 = 0 \cdot 90 v_2$	$\sec\omega + 0 \cdot 055 \tan\omega$	1·00112	1·00255	1·00428	1·00632	1·00868
$v_2 = 0 \cdot 95 v_1$	$\sec\omega - 0 \cdot 026 \tan\omega$	0·99969	0·99969	0·99999	1·00060	1·00152
$v_1 = 0 \cdot 95 v_2$	$\sec\omega + 0 \cdot 026 \tan\omega$	1·00061	1·00153	1·00275	1·00428	1·00612

In these evolutions it is supposed that $l_1 = l_2$ (see fig. 7), a condition easy to satisfy with great exactness.

It can easily be seen that all the foregoing continues to be true in the main, even if the velocity of the jet is not the same over the whole cross-section. With use of the method on jets that are not cylindrical there are some complications. Reference will only be made here to the jets investigated in this paper, the equation of which is $r = a + b \cos n\phi \,.\, \cos kz$.

When n is an even number, the oblique sections produced by the edges k can, without appreciably altering the volume of the piece cut out, be replaced by the normal sections through the points where the axis meets the oblique sections. If n is uneven, this will not be the case, but the deviation will be small for all the jets investigated in this work. The only error to be considered will thus result from the circumstance that the volume of the jet which is cut off by two normal sections, at a constant distance from each other, will vary a little with the position in relation to the stationary waves of the jet. To investigate the amount of this error the volume of the jet V_0^L between the planes $z = 0$ and $z = L$ is determined:

$$V_0^L = \int_{\phi=0}^{\phi=2\pi}\int_{z=0}^{z=L} \tfrac{1}{2} r^2 \, d\phi = \pi L \left(a^2 + \frac{b^2}{4}\right) + \pi \frac{b^2}{8k} \sin 2kL \quad . \quad . \quad . \quad . \quad (9).$$

If λ is the wave-length, then $k = 2\pi/\lambda$ and

$$V_0^L = \pi \,.\, L \,.\left(a^2 + \frac{b^2}{4}\right) + \frac{b^2}{16} \lambda \sin \frac{4\pi}{\lambda} L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$$

If λ is equal to the distance between the edges (this distance is always greater, the actual error consequently smaller than that calculated), then equation (10) shows that the greatest volume that can be cut out is

$$V_1 = \lambda\pi \left(a^2 + \tfrac{1}{4} b^2\right) + \tfrac{1}{8} \lambda b^2,$$

whilst the average value is

$$V_0 = \lambda\pi \left(a^2 + \tfrac{1}{4} b^2\right).$$

The greatest error is, expressed in per cent.,

$$\Delta = 100 \cdot \frac{\frac{1}{8}b^2}{a^2+\frac{1}{4}b^2}.$$

For

$b/a =$	0·1	0·2	0·3	0·4
$\Delta =$	0·04	0·16	0·35	0·65.

For the values of b/a here used this error is without importance.

For the "jet-catcher" used here

$$L_1 = 2{\cdot}97985 \text{ cm.}, \quad L_2 = 2{\cdot}99225 \text{ cm.}, \quad L = L_1+L_2 = 5{\cdot}9721 \text{ cm.}$$

The whole of the "jet-catcher," with the exception of the before-mentioned steel plates, is made of magnalium, which is both light and keeps in good condition. Before use the "jet-catcher" is dipped in melted paraffin, so that it is covered with a thin layer, removed only from the edges of the knives, k.

In order to prevent evaporation during weighing it is necessary to cover the openings of the "jet-catcher"; this is done by the help of two indiarubber plates fastened to two metal rods, pressed against the two openings by help of springs. It is also necessary to introduce a correction for evaporation during the cutting off, as described below.

The measuring itself takes place in the following manner: The fluid contained in the "jet-catcher" from a previous measurement is completely removed and the apparatus is carefully cleaned. The indiarubber coverings are set and the "jet-catcher" is weighed and arranged on the pendulum, whereupon the rubber covers are removed and the pendulum carried up to a horizontal position, released and allowed to complete five whole swings, after which it is caught again. As soon as this takes place the rubber covers are put on, the "jet-catcher" is taken from the pendulum and all outside drops removed, after which the weighing takes place. After the conclusion of the remaining measurements, which takes place in the course of a few minutes, the jet is stopped and the "jet-catcher" again arranged on the pendulum. The rubber covers are removed and the pendulum is allowed again to complete five whole swings, beginning with the same height as with the "cutting off," after which the rubber plates are replaced and the "jet-catcher" weighed again. If we call the weight of the liquid contained in the "jet-catcher" by the first weighing P mg., and by the second $P-p$ mg., then the loss by evaporation during these swingings is p mg. The whole weight of the quantity of the liquid cut off the jet is then $P+\alpha . p$ mg., as the loss by evaporation during the cutting off is αp mg.

The determination of the coefficient α takes place as follows: Two cuts are made, the pendulum completing only one whole swing. The quantity of liquid "cut off" is determined in the ordinary manner, after which the loss of weight, p_1, for five whole swings of the pendulum is determined as explained above. Next four cuts are made, the pendulum completing two whole swings, the corresponding loss, p_2, being determined in the same manner as above. In a similar manner p_3, p_4, and p_5 are determined. We have then

$$\alpha = \frac{\frac{3}{4}p_1+p_2+p_3+p_4+p_5}{5p_5}.$$

In Table II. are arranged the results of these determinations for water and alcohol (98·04 per cent.).

TABLE II.

	Water.	Alcohol, 98·04 per cent.
	mg.	mg.
p_1	1·75	17·0
p_2	2·30	22·0
p_3	2·35	23·4
p_4	2·40	23·6
p_5	2·45	24·0
α	0·882	0·881

In the following it is always assumed that $\alpha = 0·88$. A small error in the determination of α is of no great importance. If the worst case in this paper is taken, P is almost equal to 230 mg. and $p_5 = 25$ mg.; we have then $P+\alpha p_5 = 252$ mg. An error in α of 0·04 will give an error in the concluding weight of 1 mg. or of 0·4 per cent. The corresponding error in the surface-tension is about 0·2 per cent.

As the determination of p_5 always takes place under the same conditions—temperature, humidity, and air pressure—as those under which the "cutting off" takes place, the determination of this correction is quite certain and cannot cause great errors.

In Table III. are shown some of the values found for αp_5 corresponding to orifice No. III. All the weighings are corrected for the buoyancy of the air.

Besides the sources of error investigated there are several other circumstances that possibly could cause irregularities in the exactness of the measurements. Thus it is necessary that the vessels K of the "jet-catcher" have a certain shape, so that they can without loss receive and hold the portions of the jet cut off. With the form shown in fig. 6 I have never noticed any loss of liquid.

It is further obvious that if the speed of the "jet-catcher" when passing the jet is too slow, the disturbance in the jet produced by the first knife will have time to reach the second knife before it has cut the jet through. It is also possible that the movement in the air resulting from the movement of the pendulum and the "jet-catcher"

TABLE III.

Liquid.	αp_5.
	mg.
Water	1·8
Alcohol, 3·09 per cent. by weight	2·2
,, 9·50 ,, ,, ...	3·5
,, 46·34 ,, ,, ...	10·4
,, 74·93 ,, ,, ...	12·6
,, 81·02 ,, ,, ...	14·5
,, 90·97 ,, ,, ...	16·5
,, 98·04 ,, ,, ...	21·1
Aniline	0·6
Ammonia, density ρ15/4 = 0·9903	2·6
,, ,, = 0·9792	5·0
,, ,, = 0·9580	7·9

might have influence upon it. It is clear that both these influences are dependent upon the velocity of the "jet-catcher."

In order to investigate these questions I have made several series of experiments, one of which is given in Table IV. The result is, as can be seen, within wide limits, independent of the velocity of the "jet-catcher."

Further, I have compared the "jet-catcher" used here with another for which $L = L_1 + L_2 = 7{\cdot}9030$ cm. The difference between the results of a series of experiments on the same jet was only 0·06 per cent.

TABLE IV.—Water Jet. Velocity 273·1 cm./sec. Diameter 1·3415 mm.

Mean velocity of the "jet-catcher."	Weight for five complete oscillations of the "jet-catcher."	Deviation from mean value.	Deviation.	Corresponding deviation of the radius of the jet.
cm./sec.	mg.	mg.	per cent.	mm.
651	422·15	+0·83	+0·20	+0·00067
586	421·04	−0·28	−0·07	−0·00024
530	421·43	+0·11	+0·03	+0·00010
463	420·80	−0·52	−0·12	−0·00040
382	421·97	+0·65	+0·15	+0·00050
280	420·52	−0·80	−0·19	−0·00064
Mean value . .	421·32	Mean error . .	±0·14	±0·00047

§ 7. After the above there can hardly be any doubt that this method for the determination of the sectional area of a thin jet gives very trustworthy results, and that by this means we have a convenient method of carrying out several investi-

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

gations on such jets with an exactness that hitherto has been difficult or impossible to reach.

I will mention only very briefly some measurements of the influence of the pressure on the sectional area of a jet, keeping myself within the limits where the question is of interest for this investigation. The measurements comprise two circular apertures, No. 1 with diameter 1·514 mm. and No. 2 with diameter 0·8043 mm., both arranged as shown in fig. 10, where B is the perforated plate. For these I have determined the sectional area of water jets for heads between 50 and 100 cm. The results are shown in fig. 8, where the value *a–b* corresponds to aperture No. 1 and *c–d* to

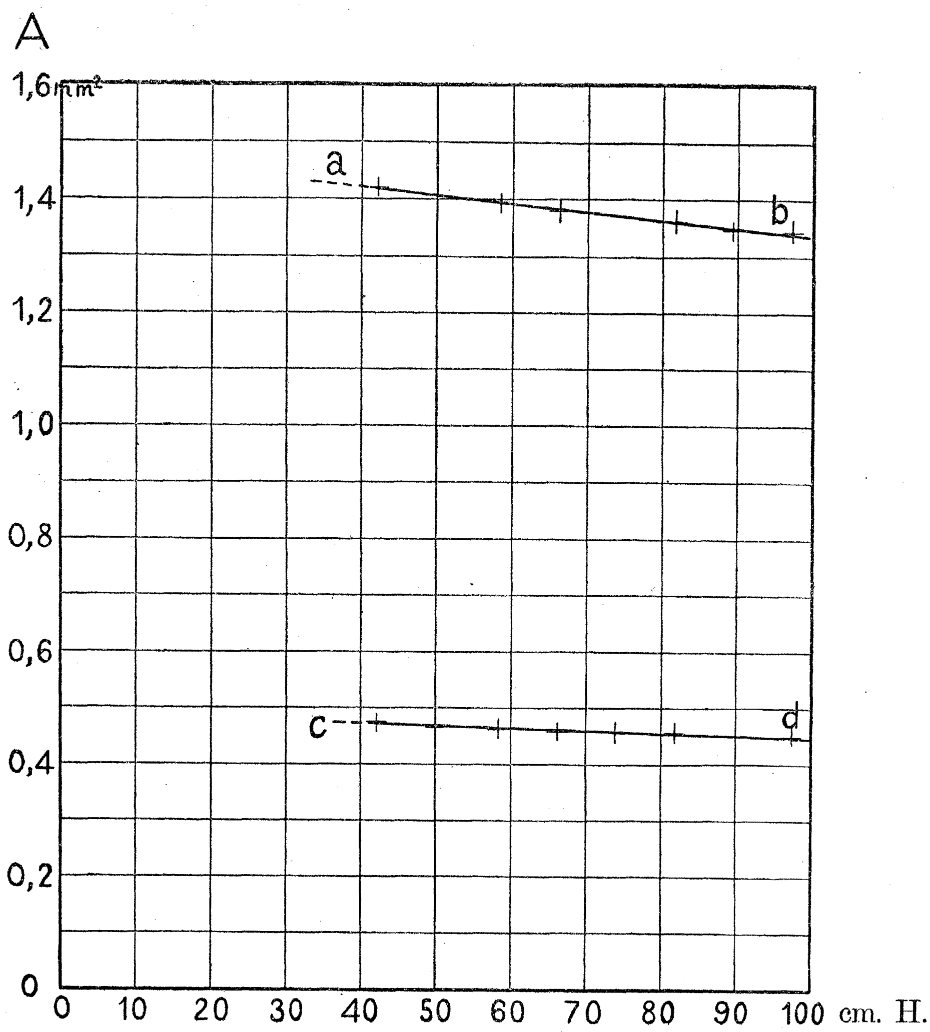


Fig. 8.

No. 2. The values measured are shown by a cross; it can be seen that they very nearly fall on the straight lines *a–b* and *c–d*.

For aperture No. 1 the sectional area decreases 4·98 per cent., while the head increases from 50–100 cm.

For aperture No. 2 the sectional area decreases 5·03 per cent., while the head increases from 50–100 cm.

The apertures used in this investigation have diameters between No. 1 and No. 2. For these, without making any great error, it can be calculated that the sectional area decreases 1 per cent. for each 10 cm. the head increases.

The determination of the discharge takes place in the usual manner and needs no comment.

Production of the desired Deviation from the Cylindrical Form of a Jet.

§ 8. In § 3 is shown the importance of the jet executing one single vibration, in other words, that its surface is determined approximately by

$$r = a+b \cos n\phi \,.\, \cos (2\pi z/\lambda_n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

as in the contrary case the determination of the wave-length λ_n causes difficulty and becomes inaccurate. By the measurements that up to now have been made with this method only little attention has been paid to this condition. Lord RAYLEIGH* writes as follows: ". . . The first set of observations here given refers to a somewhat elongated orifice of rectangular form; . . . refers to an aperture in the form of an ellipse of moderate eccentricity; . . . relate to an orifice in the form of an equilateral triangle with slightly rounded corners. . . ." PICCARD† says: "Le liquide s'écoule par un tube aplati. . . ." MEYER‡ expresses himself in the following manner on this question: "Die elliptische Oeffnung ist mittelst einer Stopfnadel durchgeschlagen, welche auf einem Oelstein solange geschliffen wurde bis eine in ein Probestück des Membran geschlagene Oeffnung die gewünschte Form und Grösse hatte. . . . Um eine grössere Genauigkeit zu erzielen, wäre vor allem auf eine schärfere Beobachtungsmetode und die Herstellung einer genau elliptischen Oeffnung Bedacht zu nehmen."

Apart from the last-mentioned reference—which according to § 3 is incorrect—the importance of, and the means for, giving the jet a single vibration has been wholly neglected.

I have endeavoured to solve this question by making the aperture as exact as possible after the formula

$$r = a+b \cos n\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

As the dimensions must be small, for example a = 0·65 mm., so as not to use too great a quantity of liquid, the work is consequently accompanied with some difficulty. The following method, however, proved itself to be good. The aperture is first drawn enlarged, fig. 9, ABCD, after which I chose a fine round file, the radius r

* RAYLEIGH, 'Roy. Soc. Proc.,' 29, p. 71, 1879 ('Papers I.,' p. 377).

† PICCARD, *loc. cit.*

‡ MEYER, *loc. cit.*

of which is somewhat smaller than the smallest radius of the curvature of the aperture. On the drawing is constructed the curve *abcd*, described by the centre of a circle with radius r rolling inside the curve ABCD. The values of the radius vector for the curve *abcd* corresponding to $\phi = 0°, 6°, 12°$, &c., are determined on the drawing. By

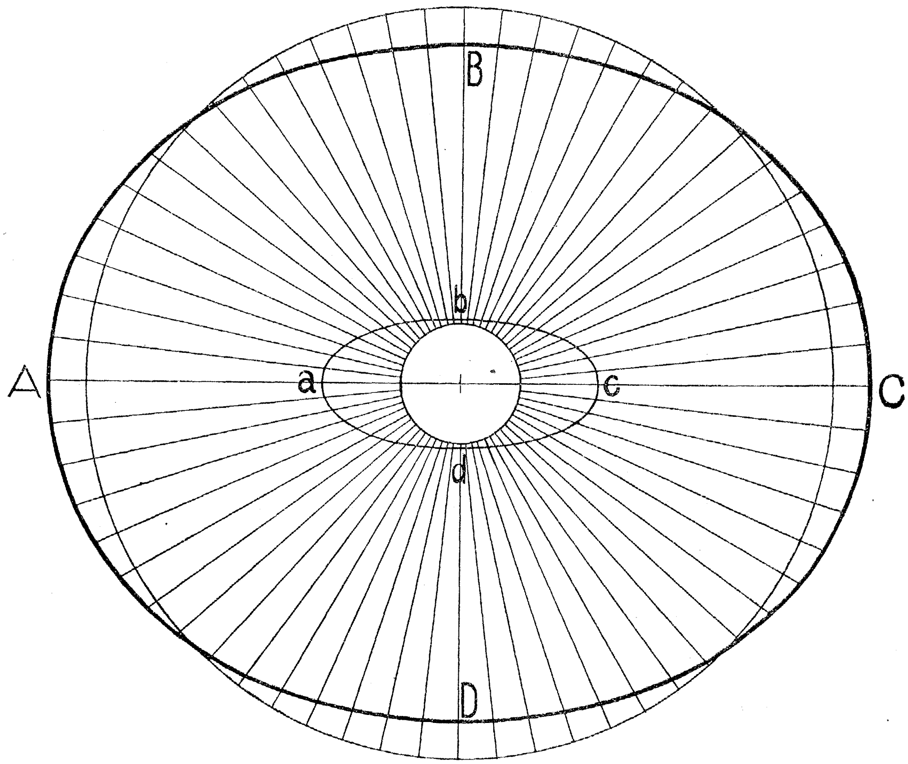


Fig. 9.

help of these values and by using a small milling machine, with the above-mentioned little file as cutter, the orifice can be cut in the correct form ABCD. Further particulars are given in the original paper.

The plate B is in most cases of platinum iridium (90 Pt+10 Ir) and has the form

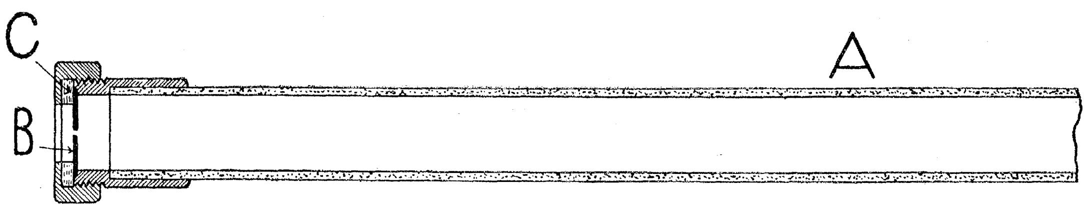


Fig. 10.

of a circular plate, about 17 mm. in diameter and about 0·5 mm. thick. In the middle of each plate the thickness is reduced to about 0·25 mm. Fig. 10 shows the ordinary arrangement of aperture and conducting tube. A represents a glass tube, B a perforated plate, and C a ring of indiarubber. Some few apertures are made in

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

brass. Microphotographs of some of the orifices used are shown on Plate 2; below each photograph is denoted the length of the largest diameter of the corresponding orifice. Further particulars will be given later.

It proved, however, that even with the best of the apertures produced in this manner the jet was not quite free from alien vibrations. That is due partly to deviations from the correct form of the aperture, but also to the fact that the cross-section of the jet is not strictly similar to the form of the aperture. This last-mentioned inconvenience would be got rid of by allowing the jet to flow out of a tube which had the correct form of cross-section. This solution is, however, for several reasons inconvenient. When the jet flows out of a tube the velocity will be less at the surface than in the axis; and, finally, the production of such a tube would be very difficult.

I have also tried to produce the deviation of the jet in another manner, namely, by using a circular orifice and a non-circular conducting tube (see fig. 11); but generally I prefer the other method.

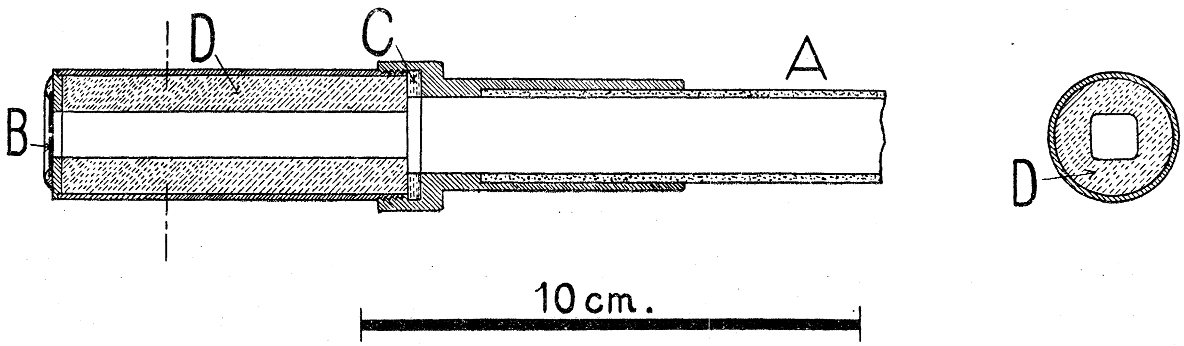


Fig. 11.

With regard to the purity of the vibrations obtained, the jet-photographs on Plates 3 and 4 will give good information. The production of these jet-photographs will be described in the next section.

Determination of the Wave-length.

§ 9. Of all the quantities on which the surface-tension according to equation [(2), § 1] depends, λ_n is undoubtedly the most difficult to determine. In all the previous measurements, as mentioned before, λ_n is determined as the distance between the summits of the jet, and the determination has taken place by direct measurement either on the jet itself or on a photograph of it. As the amplitude of the vibrations must be small, this method is very unsatisfactory and cannot give good results.

An exact determination of λ_n can be made in many ways, but they will most probably have it in common that the jet itself is used as an optical, image-forming system. Of the methods I have endeavoured to use I will only describe the following two.

The first method is illustrated by fig. 12. Here *abba* represents one of the profile lines of the jet (seen from above). L is a Nernst-lamp (1 amp. 220 volts), the linear filament being vertical. The rays coming from L are reflected by the

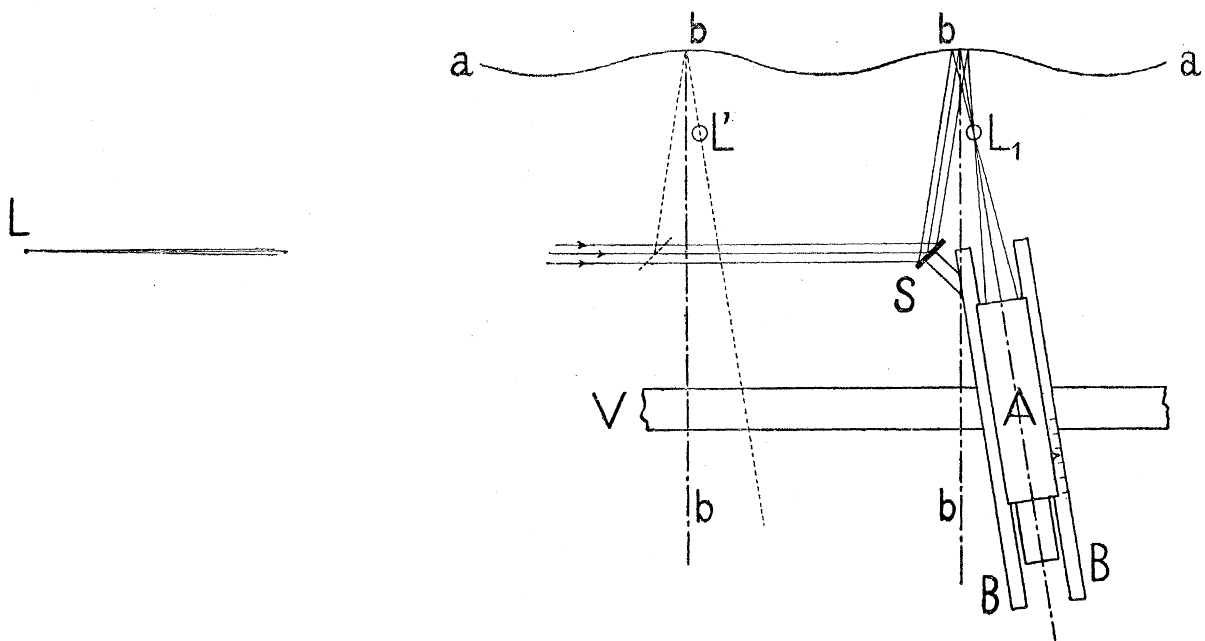


Fig. 12.

mirror S and unite after reflection from the surface of the jet in the image L_1. When the profile line is a sinusoid the distance between the images L′ and L_1 will be equal to the wave-length. This distance can be determined with great accuracy, and

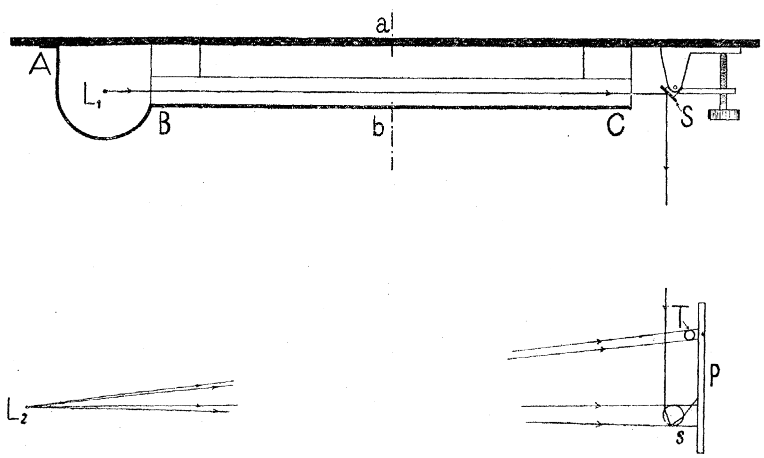


Fig. 13.

if the jet were perfectly free from alien vibrations this method would be able to give very exact results. Unfortunately it has not been possible for me to produce a jet so regular that I could make use of this method with real advantage. Even very small deviations from the desired jet-form change the position of the images very considerably. The execution of a measurement also demands much time, and this is probably the greatest drawback of the method.

The other method is as follows :—

The rays from a horizontal linear incandescent lamp L_1 (about 23 cm. long, 25 candle-power, 110 volts) is reflected from the mirror S perpendicularly down on the jet *s* (see fig. 13). Close beside the jet is arranged a vertical photographic plate P, upon which an image is formed, the approximate form of which is shown by the line *m–n* on fig. 14. The lamp L_1 is enclosed in a shield ABC.

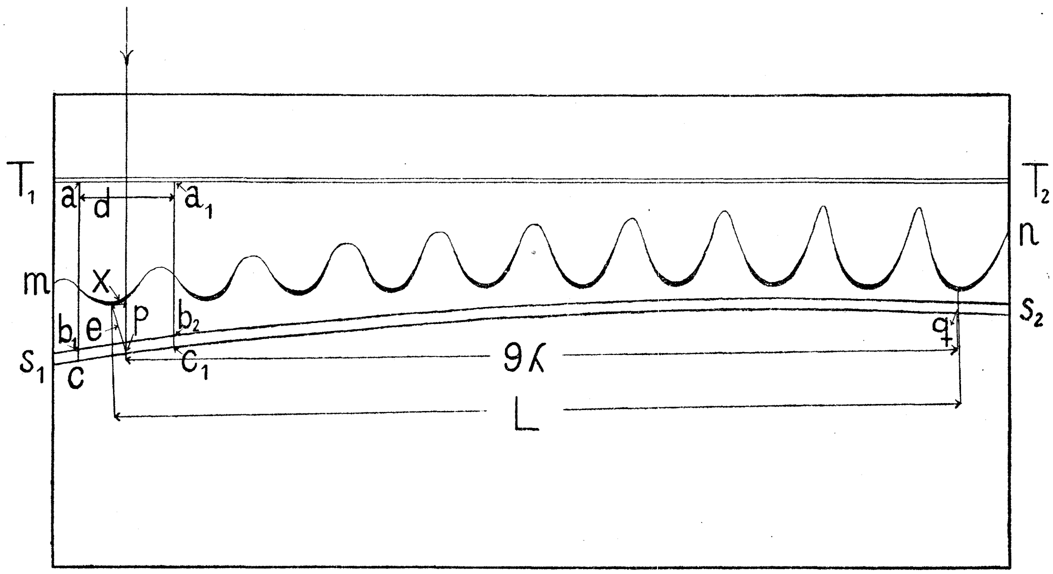


Fig. 14.

The part BC of this shield has the form shown in fig. 15, making the illumination

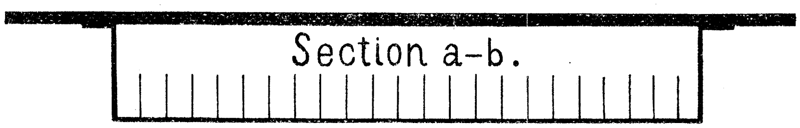


Fig. 15.

of the jet the same for its entire length. In series with the lamp L_1 another lamp, L_2, is inserted, the power of which can be regulated by the help of a rheostat placed parallel to it. L_2 is arranged at the same height as the jet, and the light from it produces a homogeneous fog on the plate P which is only interrupted by the shadows s_1–s_2 of the jet and T_1–T_2 of the wire T which is arranged horizontally in front of the plate.

The manner in which the image *m–n* (fig. 14) is formed will be explained only very briefly. In fig. 16 is shown the circular cross-section of a horizontal jet. L is a vertical ray; its direction is after two refractions and one reflection changed to L_1. The angle between L and L_1 is denoted by y and the refractive index of the liquid by n_0. With the symbols of the figure

$$y = 4b - 2i.$$

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

It is easily shown that y is maximum when

$$\sin i = \sqrt{\frac{4 - n_0}{3}}.$$

In order to illustrate the positions of the emergent rays, fig. 17 is drawn. The ray for which y is maximum cut the plate P in B, and at that point the intensity of illumination will be maximum. All the points B, collectively, form the image m–n (fig. 14).*

Let the maximum value of y corresponding to z† be denoted by y_z, then $y_z - y_0$ has the same sign as $b_n \cos kz$. The wave form of the image m–n is produced in this manner.

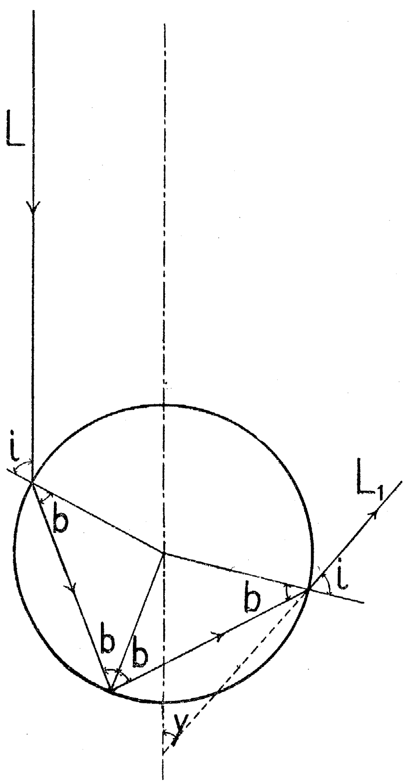


Fig. 16.

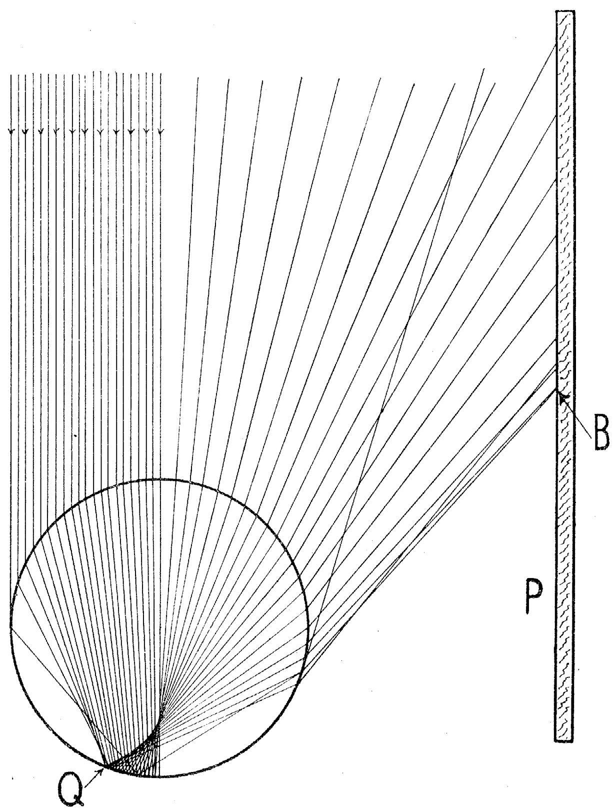


Fig. 17.

As the amplitude of the curve m–n is much greater than that of the jet, it is much easier to determine the wave-length by measuring on the image m–n than on the jet itself.

The measuring of the wave-length takes place in the following manner (fig. 14): The distance L between two homologous suitable points on the image m–n is determined. By dividing this length by the number ν of waves between the points

* For further information about this question see J. M. PERNTHER 'Meteorologische Optik' (Wien and Leipzig, 1902), p. 482.

† In $r = a + b_n \cos n\phi \cdot \cos kz \ldots$ [(1), § 1].

measured, we have λ_n. This would be perfectly correct if the jet were horizontal over the whole length, in other words, parallel to T_1–T_2, and perpendicular to the light incident on the jet. As this is not the case, the following correction is necessary: L represents the distance between the homologous points in the image, but what is in reality necessary to be known is the distance between the corresponding points p and q on the jet itself. With the symbols used in the figure, this correction for the point p will be with sufficient exactness: $x = \frac{e}{d}[ab+ac-(a_1b_1+a_1c_1)]$.

Here e is the distance from the point on the image to the point Q (see fig. 17) where the ray of minimum deflection is reflected. This correction is calculated for both the two points p and q, and the distance L_1 between these points is $L_1 = L-x-x_1$.

This formula is not quite correct, as e as a rule will have different values at the two ends, but the corresponding error is only small, and will be neglected here. L_1 is here determined as the distance between the points p and q, although in reality it is the length of the portion $p-q$ of the jet that is needed; but this error is only small for the jets examined here.

The wave-length is therefore determined by $\lambda_n = L_1/\nu$.

In the following, the wave-length is always determined by the last-mentioned method, although perhaps it is not so exact as the first, in principle; it has nevertheless great advantages compared with it. Among these advantages is the comprehensive view of the whole jet, tending to prevent mistakes, and the much shorter time needed for the determination, inasmuch as the actual measuring work can be done afterwards on the finished plate. Finally, the exactness that is reached is certainly as great as is possible, so long as it is not feasible to obtain absolutely pure jet-vibrations. One fault, however, with this method is that it is only available for transparent liquids.

During exposure the plate P is arranged in a plate-holder which is fixed in a vertical frame. This can be laid down in a horizontal position by turning the pivots below. The frame is arranged on a horizontal slide that can move in a direction at right angles to the jet. The movement of the slide towards the jet is stopped by an adjustable stop leaving a distance of about 4 mm. between the plate and the jet.

On Plates 3 and 4 are shown some photographs of jets taken in this manner; further details will be given later.

By the use of nearly monochromatic illumination still better jet-images may be obtained.

Investigations on the Influence of the Amplitude of Vibration.

§ 10. If the jet's cross-section is determined by the equation

$$r = a+b\cos n\phi \quad . \quad . \quad . \quad (1),$$

then $r_{max} = a+b$ and $r_{min} = a-b$.

In the following the amplitude of the jet will be denoted by δ, determined by

$$\delta = 100 \cdot \frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

or, according to equation (1),

$$\delta = 100 \cdot \frac{b}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2').$$

The same notation will be used for the orifices.

In § 1 the necessity of an investigation respecting the influence of the amplitude on the period of vibration has been already emphasized. The only material that is available in this case consists of the measurements made by Lord RAYLEIGH* and recorded in his original paper. For some of these I have calculated the surface-tension according to formula [(2), § 1], and arranged Table V. in order of decreasing amplitude.

TABLE V.

Orifice.	T.
Triangle with slightly rounded corners. (Table V., by RAYLEIGH) . .	dyne/cm. 57·4
Square. (Table VII., by RAYLEIGH)	64·6
Ellipse of moderate eccentricity ($\delta > 9$). (Table II., by RAYLEIGH) . .	69·5
Ellipse ($\delta = 9$). (Table IV., by RAYLEIGH)	72·9

From the values of T it is evident that the amplitude for the three jets has been too great. How far this is also the case with the jet corresponding to the orifice for which $\delta = 9$ cannot be determined on the basis of the investigations mentioned. In the measurements of MEYER† and PICCARD,‡ the amplitudes in accordance with the above have been much too great.

In order to decide this question, I have made a series of measurements with jets of water; the orifices used for these are recorded in Table VI.

The results of these experiments are found in Table VII., where the orifices are arranged with decreasing amplitudes. In the table, T indicates the surface-tension calculated upon the supposition that the amplitudes could be considered as extremely small.

* RAYLEIGH, 'Roy. Soc. Proc.,' 29, p. 71, 1879 ('Papers I,' p. 377).

† MEYER, *loc. cit.*

‡ PICCARD, *loc. cit.*

TABLE VI.

Orifice No.	Diameter of orifice. Largest.	Diameter of orifice. Smallest.	n.	δ.	Material of perforated plate.
	mm.	mm.			
I.	1·671	1·009	2	24·7	90 Pt + 10 Ir
II.	1·545	1·281	2	9·3	90 Pt + 10 Ir
III.	1·424	1·303	2	4·4	90 Pt + 10 Ir
IV.	0·830	0·820	2	0·61	90 Pt + 10 Ir
V.	1·386	1·384	2	0·14	90 Pt + 10 Ir
VII.	1·556	1·530	2	0·86	90 Pt + 10 Ir
A	2·202	—	3	—	brass
B	2·372	2·128	4	—	,,
C	2·398	2·318	6	—	,,

TABLE VII.—Ordinary Tap-Water.

$$\rho_{15/4} = 0{\cdot}99913, \quad \frac{d\mathrm{T}}{dt} = -0{\cdot}151.$$

Experiment No.	Orifice. No.	Orifice. n.	Orifice. δ.	Discharge of the jet.	Sectional area of the jet.	Wave-length.	T_t.	t.	$[\epsilon]$.	T_{15}.
				cm³./sec.	cm².	cm.	dyne/cm.		dyne/cm.	dyne/cm.
18	I.	2	24·7	2·9508	0·01060	1·2100	65·63	16·4	0·00	65·85
17	II.	2	9·3	3·1864	0·01149	1·2306	70·99	17·0	0·00	71·29
28	II.	2	9·3	4·6535	0·01097	1·8210	72·38	14·3	0·00	72·27
19	III.	2	4·4	2·9942	0·01073	1·1506	74·02	14·6	0·00	73·96
29	III.	2	4·4	4·3291	0·01020	1·7041	74·07	14·4	0·00	73·97
37	III.	2	4·4	3·8649	0·01043	1·5095	73·99	14·5	0·06	73·85
38	III.	2	4·4	3·8327	0·01043	1·4932	74·33	15·1	0·07	74·27
39	III.	2	4·4	3·8597	0·01049	1·5053	74·18	15·4	0·06	74·18
23	VII.	2	0·86	3·8745	0·01394	1·3945	74·40	15·2	0·00	74·43
33	VII.	2	0·86	5·6274	0·01334	2·0675	74·53	14·5	0·00	74·45
20	IV.	2	0·61	1·2580	0·004728	0·5800	75·03	14·8	0·00	75·00
30	IV.	2	0·61	1·8440	0·004551	0·8815	74·29	14·3	0·00	74·18
21	V.	2	0·14	3·0412	0·01097	1·1612	74·12	14·9	0·00	74·11
31	V.	2	0·14	4·4577	0·01060	1·7387	74·03	14·0	0·00	73·88
24	A	3	—	8·2429	0·02912	1·2620	71·03	15·0	0·00	71·03
25	B	4	—	8·8476	0·03113	0·8222	73·40	14·9	0·00	73·39
26	C	6	—	9·3313	0·03312	0·4457	74·83	14·2	0·00	74·70

The values of T for the orifices A, B, and C agree very well with those for the apertures for which $n = 2$, when due regard is taken to the amount of the amplitude. In the following, notice is only taken of those orifices for which $n = 2$.

TABLE VIII.

Orifice.	δ.	Mean value of T_{15}.	$74 \cdot 34 - T_{15}$.	$\frac{74 \cdot 34 - T_{15}}{\delta^2}$.	$0 \cdot 02 . \delta^2$.
		dyne/cm.	dyne/cm.		
I.	24·7	65·85	+8·49	0·0139	12·2
II.	9·3	71·78	+2·56	0·0292	1·73
III.	4·4	74·05	+0·29	0·0155	0·39
VII.	0·86	74·44	−0·10	—	0·015
IV.	0·61	74·59	−0·25	—	0·008
V.	0·14	74·00	+0·34	—	0·000
VII., IV., V.	—	74·34	—	—	—

In Table VIII. are given the mean values of T_{15} corresponding to each aperture. It is evident that the orifices I. and II. have too great amplitudes. For the orifice III. it cannot with certainty be determined on the basis of Table VIII.; if the amplitude has any influence on the period it can only be said that the influence must be small. The amplitude for the next orifice, VII., is only one-fifth of that of orifice III., and there can therefore be no doubt that the amplitude of orifice VII. is sufficiently minute. That is still more certain for the orifices IV. and V. In Table VIII. is also inserted the mean value of T_{15} for holes VII., IV., and V. taken together. This mean value is 74·34. The fourth column contains the values of $74 \cdot 34 - T_{15}$ and the fifth column contains the values of $\frac{74 \cdot 34 - T_{15}}{\delta^2}$ for the orifices I., II., and III. Finally in the last column the values of $0 \cdot 02 . \delta^2$ are given for all the orifices. The numbers in the last two columns must naturally be taken with all possible reserve, but still they serve to explain, and at the same time also prove, the correctness of the above result, namely, that the amplitudes for holes VII., IV., and V. are so small that their values have no appreciable influence on the determination of the surface-tension.

Reference has only been made in the above to the amplitude of the aperture and

TABLE IX.

Orifice.	Pressure.	δ measured on the jet.
	cm.	
I.	95	37·6
I.	43	29·2
II.	95	16·3
III.	43	4·9
VII.	43	1·06
IV.	43	0·88

not to the jet itself. To get an idea of the size of the amplitude of the jet I have taken photographs of different jets, and afterwards by help of an object-micrometer measured the largest and smallest diameters at a distance of 3 to 4 cm. from the apertures. The results of these determinations are given in Table IX.; it appears that the amplitude of the jet is somewhat greater than the amplitude of the orifice, and that it increases with the pressure.

Therefore the above conclusion respecting the permissible amplitude will not always hold good. The pressure, about 70 cm., used by the measurements made here, lies, however, within the limits investigated in Table VII. (about 42 up to 97 cm. pressure).

The nature of the liquid may also play a part; but that has hardly any great influence as far as these investigations are concerned.

In order to attain greater certainty on this point I have determined the surface-tension of some other liquids by measurements with orifices III., V., and VII.; the results obtained are shown in Table X. This is calculated as follows: In the same manner as in Table VII., T_{15} is determined for each individual orifice, and for each of these the mean value is taken. The mean value for all the measurements with apertures VII. and V. is taken as the correct value of the surface-tension for the liquid in question. Table X. contains the deviations from the mean value found in this manner, with reference to the orifices in question, shown as a percentage of the surface-tension.

TABLE X.

Liquid.	Orifice III.	Orifice V.	Orifice VII.
	per cent.	per cent.	per cent.
Ordinary tap-water	−0·23	−0·31	+0·30
Distilled water	−0·54	−0·44	+0·44
$CuSO_4 + Aq$; $\rho = 1·0506$	−0·34	−0·17	+0·19
5·79 per cent. alcohol + 94·21 per cent. water	−0·90	−0·07	+0·00
Mean value	−0·50	−0·25	+0·23

It appears that the results for orifice VII. are generally a little larger than for orifice V. The difference lies, however, within the limit of error, as the determination for orifice V. is difficult. It proves, however, that under the present conditions the amplitude for orifice VII. is sufficiently small. On the other hand, however, it also appears that the values for orifice III. are, as a rule, a little too small. It would therefore be natural to carry out the measurements with orifices VII. and V. The determination is, however, in reality made with orifices VII. and III., for the following reason: The amplitude for orifice V. is so small that the determination of the wave-length is difficult and uncertain, especially for liquids with small surface-tension, or, in other words, large wave-lengths. On the other hand, the measurements of the wave-length for orifice III. are carried out with great exactness and easiness, just as the determination with orifice VII. is, as a rule, quite good. I have therefore chosen these two orifices and corrected for the too great amplitude of orifice III. by adding 0·5 per cent. to the results, a correction obtained from Table X. In the following this correction is represented by [δ].

Execution of Observations.

§ 11. All the experiments to No. 37 are made with ordinary tap-water under constant pressure, produced in the manner explained in § 4. In all the other experiments the pressure diminished as the liquid ran out. This variation was, however, only small, as the cross-section of the reservoir used was about 400 cm^2, and the quantity of liquid used about 1000 cm^3. The pressure for all the experiments after No. 40 was about 70 cm.

The measurements themselves took place in the following manner:—

The orifice is closed by a wooden plug. The requisite quantity of liquid is poured into the reservoir, care being taken to fill both the conducting tube and the jet tube completely, the plug is withdrawn and the jet started. Then the jet is adjusted so as to be parallel to the plate-holder and to have the height suitable for the "jet-catcher." The last adjustment is easily controlled by the shadow of the jet on a ground glass placed in the frame and furnished with marks, between which the shadow must fall. At the same time the jet tube is moved until the image (*m–n*, fig. 14) of the jet is as sharp and clear as possible. In order to enable these adjustments to be made easily, the jet tube is arranged in a bridge that can be moved in all directions by help of screws.

As the direction of the "jet-catcher" once and for all is parallel to the frame, the jet by the above adjustments is brought into the required position and the measuring of its cross-section can take place. Immediately afterwards the measurements of the discharge begin, after which the plate-holder with an unexposed plate is placed in the frame, which is lying down. The light is then shut off, the shutter removed from the plate-holder and the slide moved into its position, whereupon the frame is brought up to its vertical position and the lamps L_1 and L_2 lighted. After exposing for about 15 seconds the lamps are turned out, the slide brought back and the shutter replaced in the plate-holder. The entire photographing process takes about 40 seconds. Before finishing the measurement of the discharge another photograph is taken in the same manner.

Finally the necessary weighing takes place and the evaporation is determined as explained in § 6.

In changing from one liquid to another the whole apparatus is cleaned very carefully and finally washed out with distilled water, after which it stands for some time to dry. Before use it is washed out with some of the liquid to be tested.

Various Remarks.

§ 12. On account of the pressure not being constant, it is necessary to investigate the influence of its variations.

According to § 7 the cross-section of the jet increases about 1 per cent. for every 10 cm. the pressure decreases. The cross-section ought, then, to be measured at mean pressure, but is in fact determined at the commencement of the experiment. If the liquid pressure has diminished h cm. during the experiment, the mean cross-section is

$$A + \frac{h}{2000},$$

where A is the measured cross-section. The corresponding correction $[\epsilon]$ in the surface-tension T is, then, with sufficient exactness

$$[\epsilon] = T \cdot \frac{h}{4000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

This correction is always negative.

Influence of the Variation of Pressure on the Wave-Length and Discharge.—If the effective pressure reduces from H cm. to $H-h$ cm., the first photograph of the jet will correspond to the pressure $H-y$ cm. and the last with sufficient exactness to the pressure $H-h+y$ cm. The corresponding velocities are

$$V_a = \sqrt{2g(H-y)}\,; \qquad V_b = \sqrt{2g(H-h+y)}.$$

The mean value is

$$V_{ab} = \tfrac{1}{2}\left[\sqrt{2g(H-y)} + \sqrt{2g(H-h+y)}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

As y and h are small compared with H, this expression can without any appreciable error be reduced to

$$V_{ab} = \tfrac{1}{2}\left[\sqrt{2gH} + \sqrt{2g(H-h)}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2').$$

As the wave-length is determined as the mean of the results from the two plates, V_{ab} is the velocity corresponding to the wave-length measured. The average velocity V_0 that determines the discharge Q is, as is known, similarly determined by

$$V_0 = \tfrac{1}{2}\left[\sqrt{2gH} + \sqrt{2g(H-h)}\right].$$

In this manner no correction is demanded on account of variable pressure in the determination of the wave-length and the discharge.

The curvature of the jet produces a small error since the cross-section is determined for the highest part of the jet. The average cross-sectional area will therefore be a little smaller than that measured. Under the conditions used here this error will only be insignificant.

The influence of the viscosity is, according to equation [(4), § 1], determined by the coefficient

$$g = 1 + \frac{\delta^2}{4\pi^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where δ is the logarithmic decrement of the vibration. The determination of δ is made by measuring on the image (m–n, fig. 14) of the jet, making the supposition that the amplitudes of the image are proportionate to those of the jet. The results are—

For water :	$\delta = 0{\cdot}074$; $g = 1{\cdot}00014$;
,, 98·04 per cent. alcohol + 1·96 per cent. water :	$\delta = 0{\cdot}173$; $g = 1{\cdot}00076$;
,, 46·34 per cent. alcohol + 53·66 per cent. water :	$\delta = 0{\cdot}210$; $g = 1{\cdot}00110$;
,, aniline :	$\delta = 0{\cdot}265$; $g = 1{\cdot}0018$.

The numbers given are only approximate, but they show that g, in all cases considered here, is almost equal to 1, and as the determination of δ is uncertain, no correction is introduced. An exact investigation of the influence of the viscosity on the form of the jet image cannot be made until the theory of this image is further developed.

Remarks on the Jet Photographs.

§ 13. On Plates 3 and 4 are shown fifteen jet photographs. In each photograph the number of the orifice and the nature of the liquid are denoted.

The photographs are arranged according to the orifices, in the same order as in Table VII., and the remarks concerning them are given in the same order.

Plate 3, figs. 6, 7 and 8, shows clearly the influence of viscosity on the damping of the vibration. Fig. 6 is a water jet : this has only a very small damping. Fig. 8 is an alcohol jet : with this the damping is a little larger. Fig. 7 is a jet of a mixture of water and alcohol (46·34 per cent. alcohol + 53·66 per cent. water) : with this the damping is much greater than for the other two. The viscosity has about the following values in these three cases (TH. GRAY, 'Physical Tables,' Table 151, 1897) :—

, 0·012 by 15° C. ; , 0·036 by 15° C. ; , 0·014 by 15° C.

As the logarithmic decrement is the same for all vibrations, the fundamental vibration will be purer at some distance from the orifice than immediately after the jet has been formed, as is also shown on several of the photographs.

It appears from these photographs that the jet image is very well adapted to investigation of the jet vibrations. Similarly, they show that it is possible to make orifices which for all practical purposes are correct. In further investigations by this method it will be possible to go still farther in this direction.

All the jet photographs commence about 1·5 cm. from the orifice.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

RESULTS.

Water.

§ 14. For the determination of the surface-tension of water I have made three series of measurements, the results of which are shown in the Tables XI., XII., and XIII.

The first table refers to ordinary tap-water, and gives a mean value

$$T_{15} = 74{\cdot}33 \text{ dyne/cm.}$$

The second table is for freshly distilled water, and gives

$$T_{15} = 74{\cdot}31 \text{ dyne/cm.}$$

The third table is for distilled water that has been kept for about a year in a stoppered bottle: the result is

$$T_{15} = 74{\cdot}23 \text{ dyne/cm.}$$

The greatest value found is in experiment No. 44 (Table XIII.):—

$$T_{15} = 74{\cdot}80 \text{ dyne/cm.}$$

The least is in experiment No. 41 (Table XIII.), namely:—

$$T_{15} = 73{\cdot}40 \text{ dyne/cm.}$$

The greatest deviation in the 18 experiments recorded in the Tables XI.-XIII. is thus about 1·9 per cent.

TABLE XI.—Ordinary Tap-Water.

$$\rho_{15/4} = 0{\cdot}99913, \quad \frac{dT}{dt} = -0{\cdot}151.$$

Experiment No.	Orifice.	Discharge of the jet.	Sectional area of the jet.	Wave-length.	T_t.	t.	$[\epsilon]$.	$[\delta]$.	T_{15}.
		cm³./sec.	cm².	cm.	dyne/cm.	° C.	dyne/cm.	dyne/cm.	dyne/cm.
19	III.	2·9942	0·01073	1·1506	74·02	14·6	0·00	0·37	74·33
29	III.	4·3291	0·01020	1·7041	74·07	14·4	0·00	0·37	74·35
37	III.	3·8649	0·01043	1·5095	73·99	14·5	0·06	0·37	74·22
38	III.	3·8327	0·01043	1·4932	74·33	15·1	0·07	0·37	74·65
39	III.	3·8597	0·01049	1·5053	74·18	15·4	0·07	0·37	74·54
23	VII.	3·8745	0·01394	1·3945	74·40	15·2	0·00	—	74·43
33	VII.	5·6274	0·01334	2·0675	74·53	14·5	0·00	—	74·47
21	V.	3·0412	0·01097	1·1612	74·12	14·9	0·00	—	74·12
31	V.	4·4577	0·01060	1·7387	74·03	14·0	0·00	—	73·88
				Mean value of orifice III.					74·42
				,, ,, ,, VII.					74·45
				,, ,, ,, V.					74·00
				,, ,, all experiments					74·33

TABLE XII.—Distilled Water.

Tested about two days after the distillation.

$$\rho_{15/4} = 0{\cdot}99913, \quad \frac{dT}{dt} = -0{\cdot}151.$$

Experiment No.	Orifice.	Discharge of the jet.	Sectional area of the jet.	Wave-length.	T_t.	t.	$[\epsilon]$.	$[\delta]$.	T_{15}.
		cm³/sec.	cm².	cm.	dyne/cm.	° C.	dyne/cm.	dyne/cm.	dyne/cm.
129	III.	3·8514	0·01042	1·5038	74·07	13·3	0·06	0·37	74·12
130	III.	3·8516	0·01043	1·5000	74·41	13·6	0·06	0·37	74·51
131	VII.	5·0647	0·01381	1·8424	74·36	14·0	0·06	—	74·15
132	VII.	5·0696	0·01380	1·8408	74·64	14·2	0·06	—	74·45
Mean value of orifice III.									74·32
,, ,, ,, VII.									74·30
,, ,, all experiments									74·31

TABLE XIII.—Distilled Water.

Tested about one year after the distillation. In the meantime kept in a corked vessel.

$$\rho_{15/4} = 0{\cdot}99913, \quad \frac{dT}{dt} = -0{\cdot}151.$$

Experiment No.	Orifice.	Discharge of the jet.	Sectional area of the jet.	Wave-length.	T_t.	t.	$[\epsilon]$.	$[\delta]$.	T_{15}.
		cm³./sec.	cm².	cm.	dyne/cm.	° C.	dyne/cm.	dyne/cm.	dyne/cm.
40	III.	3·7997	0·01037	1·4869	73·83	19·0	0·06	0·37	74·74
41	III.	3·7450	0·01032	1·4808	72·50	18·9	0·06	0·37	73·40
42	V.	3·9079	0·01079	1·5224	73·05	18·6	0·06	—	73·53
43	V.	3·8899	0·01074	1·5068	74·03	18·2	0·06	—	74·45
44	VII.	4·9899	0·01354	1·8226	74·39	18·1	0·06	—	74·80
45	VII.	4·9532	0·01358	1·8115	74·10	18·0	0·06	—	74·49
Mean value of orifice III.									74·07
,, ,, ,, V.									73·99
,, ,, ,, VII.									74·64
,, ,, all experiments									74·23

That the results are almost the same for the three kinds of water is not surprising, as DUPRÉ* and Lord RAYLEIGH† have shown that even very considerable impurities

* DUPRÉ, 'Théorie mécanique de la chaleur,' p. 376, Paris, 1869.

† RAYLEIGH, 'Roy. Soc. Proc.,' 47, p. 281, 1890 ('Papers III.,' p. 340).

do not appreciably alter the surface-tension, as far as quite fresh surfaces are concerned.

As the result of my experiments I fix the initial value of the surface-tension of water as

$$T_{15} = 74\cdot30 \text{ dyne/cm.}$$

The surface-tension of water has been so often determined, and in so many manners, that even a moderately exhaustive representation of the results is impossible and, besides, without great interest, as many of the measurements have but little value. In Table XIV. are shown the results of a few determinations by the capillary-tube method, and Table XV. contains most of the results obtained by the method of capillary ripples.

With respect to the values found by the capillary-tube method, it appears that, with the exception of QUINCKE's values, they are all smaller than those found here. This is quite natural, because it is the stationary value of the surface-tension that is measured by the capillary-tube method. Under the given conditions this value must be smaller than the initial value.

The values found by the method of capillary ripples in most cases agree well with the value found here. They are as follows:—Lord RAYLEIGH, 74·35 dyne/cm.; DORSEY, 73·72; WATSON, 74·76; and KALÄHNE, 74·22 dyne/cm. The mean value for all four is

$$T_{15} = 74\cdot26 \text{ dyne/cm.}$$

An exception from this, however, is made by GRUNMACH's measurements (BRÜMMER and LOEWENFELD, who worked exactly in the same manner as GRUNMACH, are not mentioned here).

GRUNMACH's measurements divide themselves into two groups, the surface of the liquid being either the same during the investigation or continually renewed. Table XV. shows that GRUNMACH's value for distilled water in the first case is

$$T_{15} = 78\cdot41 \text{ dyne/cm.,}$$

and in the other

$$T_{15} = 75\cdot89 \text{ dyne/cm.}$$

These results are very extraordinary, for two reasons. Firstly, it would be expected that the former value would agree with the values found with the same method by other investigators. This, however, is far from being the case, as GRUNMACH's value, 78·41 dyne/cm., is 5·6 per cent. larger than the corresponding mean value, 74·26 dyne/cm., of the other measurements.

Secondly, it would be supposed that the surface-tension for the continually renewed surface would be the greater. GRUNMACH, however, came to the opposite result and found a value 3·3 per cent. lower in this case.

A satisfactory explanation of this circumstance will certainly demand fresh investigations, and before these are finished it will be difficult to judge of the value of

TABLE XIV.—The Surface-tension of Water. Method of Measurement, Capillary Tubes.

Authority.	Publication.	The surface in contact with	t.	T_t.	T_{15}.	Remarks.
			° C.	dyne/cm.	dyne/cm.	
VOLKMANN	'WIED. Ann.,' 56, p. 457, 1895	Damp air	15·0	73·26	73·26	VOLKMANN'S values for temperatures from 0° to 40° C. are generally accepted as the most exact by this method of measurement.
QUINCKE	'WIED. Ann.,' 52, p. 1, 1894	Damp air	18	72·4 to 76·9	72·9 to 77·4	Of QUINCKE'S many publications I have here only quoted that containing his latest determinations.
DOMKE	'Wiss. Abh. d. K. Norm.-Aich.-Komm.,' Heft III., 1902	Damp air	18·2	72·98	73·46	
WEINSTEIN	'Metronomische Beiträge,' No. 6, 1889	Damp air	20·0	71·1	71·86	
RAMSAY and SHIELDS	'Zeitschr. phys. Chem.,' 12, p. 471, 1893	Saturated vapour	20·0	70·6	71·36	The water was tested in sealed glass tubes.
SENTIS	'Journ. d. Phys.,' 6, p. 183, 1897	Damp air	20·0	73·0	73·76	
RODENBECK	'Diss.,' Bonn, 1879, Beibl. 4, p. 104	Damp air	17·5	71·7	72·08	
ROTHER	'WIED. Ann.,' 21, p. 576, 1884	Damp air	15·0	72·02	72·02	Tested by tubes of elliptical cross-section.
GOLDSTEIN	'Zeitschr. phys. Chem.,' 5, p. 233, 1890	Damp air	22·0	72·2	73·26	

Table XV.—The Surface-tension of Water. Method of Measurement, Capillary Ripples.

Authority.	Publication.	The surface in contact with	t.	T_t.	T_{15}.	Remarks.	
			° C.	dyne/cm.	dyne/cm.		
Rayleigh	'Phil. Mag.,' 30, p. 386, 1890 ('Papers III.,' 391)	Damp air	18	73·9	74·35	Distilled water and ordinary tap-water without appreciable difference.	
Dorsey	'Phil. Mag.,' 44, pp. 134, 369, 1897	Damp air	18	73·17	73·72		
Watson	'Phys. Rev.,' 12, p. 257, 1901	Damp air	20	74·0	74·76		
A. Kalähne	'Ann. d. Phys.,' 7, p. 440, 1902	Damp air	18	73·62	74·07	Water in a glass dish filled without overflowing.	Kalähne used the rippled surface as a diffraction grating.
A. Kalähne	'Ann. d. Phys.,' 7, p. 440, 1902	Damp air	18	73·81	74·26	Water in a glass dish filled to overflowing.	(as above)
A. Kalähne	'Ann. d. Phys.,' 7, p. 440, 1902	Damp air	18	74·14	74·59	Water in a nickel dish.	(as above)
A. Kalähne	'Ann. d. Phys.,' 7, p. 440, 1902	Damp air	18	73·77	74·22	Kalähne's mean value.	(as above)
L. Grunmach	'Wiss. Abh. d. K. Norm.-Aich.-Komm.,' Heft III., 1902	Damp air	16·9	78·12	78·41	Distilled water in a porcelain dish without renewal of the surface. Grunmach's values differ up to 16 per cent. from each other (see Experiments 30 IV. and 7 V., p. 152).	
L. Grunmach	'Wiss. Abh. d. K. Norm.-Aich.-Komm.,' Heft III., 1902	Damp air	19·1	75·27	75·89	Distilled water with surface renewal. Greatest deviation between his results is 3·9 per cent.	
A. Brümmer	'Diss.,' Rostock, 1903	Damp air	15	74·92	74·92	Grunmach's method with surface renewal.	
Kolowrat-Tscherwinski	'J. d. russ. phys. chem. Ges.,' 36, 1904	Damp air	0	75·02	72·77		
K. Loewenfeld	'Diss.,' Berlin, 1905	Damp air	15	75·30	75·30	Grunmach's method with surface renewal.	

GRUNMACH's results. I cannot, however, omit to draw attention to the fact that the mutual agreement between GRUNMACH's values is only small. Within the first group of measurements the deviation amounts up to 16 per cent., and for the last to 3·9 per cent.

The lowest of GRUNMACH's values in the two cases are

$$T_{15} = 70{\cdot}4 \text{ dyne/cm.} \quad \text{and} \quad T_{15} = 74{\cdot}2 \text{ dyne/cm.}$$

Both are lower than the values found here.

Toluol.

The value found according to Table XVI. is

$$T_{15} = 28{\cdot}76 \text{ dyne/cm.,}$$

in complete agreement with VOLKMANN's results, namely,

$$T_{15} = 28{\cdot}79 \text{ dyne/cm.}$$

This was to be expected, as in this case the initial and the stationary value of the surface-tension must be very nearly equal.

Aniline.

The liquid used was marked "pure," but was, however, a little coloured.

The result of the measurements is shown in Table XVI. The value is

$$T_{15} = 43{\cdot}00 \text{ dyne/cm.}$$

The corresponding value by VOLKMANN is

$$T_{15} = 44{\cdot}30 \text{ dyne/cm.,}$$

also considerably larger. To this may be remarked that VOLKMANN himself estimates his determination as somewhat uncertain, and, further, that aniline is somewhat soluble in water, so that there has possibly been formed a layer of water on its surface. Using the method of the maximum pressure of small air bubbles, FEUSTEL* found

$$T_{15} = 46{\cdot}6 \text{ dyne/cm.}$$

Aqueous Solutions of Ammonia.

Measurements were made on three solutions of ammonia. The values are (see Table XVI.),

For $\rho_{15/4} = 0{\cdot}99030$, $T_{15} = 71{\cdot}25$ dyne/cm.,

,, $\rho_{15/4} = 0{\cdot}97921$, $T_{15} = 68{\cdot}02$ dyne/cm.,

,, $\rho_{15/4} = 0{\cdot}95801$, $T_{15} = 64{\cdot}69$ dyne/cm.

* FEUSTEL, *loc. cit.*

TABLE XVI.

Liquid.	Density. $\rho_{15/4}=$	Number of measurements.	Greatest deviation from the mean value.	Mean value. $T_{15}=$
			per cent.	dyne/cm.
Toluol	0·86736	2	0·07	28·76
Aniline	1·0250	5	0·50	43·00
Aqueous solution of ammonia	0·99030	4	0·66	71·25
" " "	0·97921	4	0·75	68·02
" " "	0·95801	4	0·70	64·69
Solution of copper sulphate	1·05030	6	0·53	74·27
Diluted sulphuric acid	1·08130	4	1·30	74·89
" " "	1·14316	4	0·89	74·44
Aqueous ethyl alcohol—				
1·13 per cent. alcohol + 98·87 per cent. water	0·99702	4	1·05	69·65
3·09 " " + 96·91 " "	0·99350	4	0·04	62·99
5·79 " " + 94·21 " "	0·98910	5	0·55	56·66
9·50 " " + 90·50 " "	0·98374	4	0·40	50·23
25·63 " " + 74·37 " "	0·96328	4	0·89	34·98
37·88 " " + 62·12 " "	0·94304	4	0·95	30·52
46·11 " " + 53·89 " "	0·92631	4	1·50	28·07
46·34 " " + 53·66 " "	0·92578	4	1·12	28·73
49·22 " " + 50·78 " "	0·91952	4	0·98	26·57
50·99 " " + 49·01 " "	0·91567	4	0·33	27·45
59·37 " " + 40·63 " "	0·89841	4	1·28	26·55
74·93 " " + 25·07 " "	0·86012	4	0·59	25·51
81·02 " " + 18·98 " "	0·83522	4	0·90	24·57
90·97 " " + 9·03 " "	0·81973	4	0·46	23·82
98·04 " " + 1·96 " "	0·79960	8	1·71	22·80

For comparison I will take the values found by DOMKE* by the capillary-tube method. The values of DOMKE and those of the author agree fairly well, as the last, as could be expected, are all a little larger than the first. If we compare DOMKE's values augmented with the difference between the author's and DOMKE's results for distilled water (namely, 74·30 − 73·00 = 1·30 dyne/cm.), it appears that the difference between the two sets of values is not great (see Table XVII.).

LOEWENFELD† has determined the surface-tension of ammonia by the method of capillary ripples with surface renewal. His results differ rather much from the author's.

* DOMKE, 'Wiss. Abh. d. K. Norm.-Aich.-Komm.,' Heft III., Berlin, 1902.

† LOEWENFELD, 'Diss.,' Berlin, 1905.

TABLE XVII.

$\rho_{15/4}$.	*a.* DOMKE.	*b.* DOMKE + 1·30.	*c.* The author.	*b* − *c*.
	dyne/cm.	dyne/cm.	dyne/cm.	dyne/cm.
1·000	73·0	74·3	74·30	0·00
0·9903	69·2	70·5	71·25	−0·75
0·9792	66·5	67·8	68·02	−0·22
0·9580	63·3	64·6	64·69	−0·09

Solution of Copper Sulphate.

$$\rho_{15/4} = 1·0503.$$

The mean value of the determination of the surface-tension of this solution is found in Table XVI.,

$$T_{15} = 74·27 \text{ dyne/cm.}$$

The value is practically the same as for water.

Diluted Sulphuric Acid.

Of diluted sulphuric acid two different concentrations were investigated. The mean values are (see Table XVI.)

$$\text{For } \rho_{15} = 1·0813, \quad T_{15} = 74·89 \text{ dyne/cm.,}$$
$$\text{,,} \quad \rho_{15} = 1·14316, \quad T_{15} = 74·44 \text{ dyne/cm.}$$

The corresponding values, according to GRUNMACH (with capillary-wave method with renewed surface), are

$$T_{15} = 76·5 \text{ dyne/cm.} \quad \text{and} \quad T_{15} = 77·8 \text{ dyne/cm.}$$

Aqueous Ethyl Alcohol.

DOMKE* has given a table of the results obtained by different authors for the surface-tension of absolute alcohol. The results are reduced to a temperature of 15° by the use of $dT/dt = -0·08$. The mean value of the fifteen values considered is

$$T_{15} = 23·1 \text{ dyne/cm.}$$

The lowest value is $T_{15} = 22·2$ dyne/cm., the highest is $T_{15} = 24·3$ dyne/cm.

DOMKE himself found by the capillary-tube method,

$$T_{15} = 23·0 \text{ dyne/cm.}$$

* DOMKE, 'Wiss. Abh. d. K. Norm.-Aich.-Komm.,' Heft III., Berlin, 1902.

GRUNMACH* found by the method of capillary ripples :—

For absolute alcohol that has not been in contact with the air and for a continually renewed surface	$T_{15} = 19{\cdot}6$ dyne/cm.
For absolute alcohol the surface of which had been in contact with the air for one half-hour .	$T_{15} = 21{\cdot}2$ dyne/cm.
For absolute alcohol that had for some time been continually in contact with the air	$T_{15} = 26{\cdot}3$ dyne/cm.

The author determined the surface-tension of several mixtures of alcohol and water. The mixtures investigated and the results obtained are shown in Table XVI. The value of $d\text{T}/dt$ used by the calculation of the table is determined by the following formula :—

$$\frac{d\text{T}}{dt} = -(0{\cdot}151 - p\,.\,0{\cdot}0007),$$

where p is the percentage of alcohol by weight.

The results are shown in Plate 2, fig. 1, where also the results found by B. WEINSTEIN† are shown.

With regard to the surface-tension of absolute alcohol it can pretty certainly be taken that the value determined by this method will very approximately be

$$T_{15} = 22{\cdot}5 \text{ dyne/cm.},$$

in complete disagreement with the value found by GRUNMACH, $T_{15} = 19{\cdot}6$ dyne/cm., in the case of continuous surface renewal.

How this great difference is produced must be determined by further investigations on the subject.

CONCLUDING REMARKS.

§ 15. In the use of Lord RAYLEIGH'S method for the determination of the surface-tension of liquids it is necessary to pay attention to the following remarks :—

It is necessary to determine either the velocity or the cross-section of the jet by direct measurement, as the calculation of the velocity from TORRICELLI'S formula may lead to great errors.

The greatest care must be taken to obtain jets executing one vibration only, corresponding to only one value of n.

The amplitude of vibrations must be very small.

The determination of the wave-length must be performed by suitable optical

* GRUNMACH, *loc. cit.*

† WEINSTEIN, 'Metronomische Beiträge,' No. 6, Berlin, 1889.

methods, as the smallness of the amplitudes renders the direct measurement impossible.

With due consideration of the above remarks, Lord RAYLEIGH'S method is a very good one, and is highly deserving of use in the future on account of its great fundamental advantages.

With regard to the results obtained in this investigation, the author desires to call attention to the remarkable discrepancies between his results and those of GRUNMACH, who used the method of capillary ripples with renewal of the surface (see p. 343 above). It was to be expected that the difference between the two sets of values should be small, and further, that GRUNMACH'S values should be intermediate between the author's values and those obtained by the capillary-tube method. But that is very far from being the case: the differences are rather great, and of a sign opposite to that expected.

In further application of this method the author would propose to use apertures with amplitudes between 4·0 and 0·5; for instance, the following set:—

$$\delta = 4{\cdot}0,\ \delta = 2{\cdot}0,\ \delta = 1{\cdot}0,\ \text{and}\ \delta = 0{\cdot}5.$$

TABLE XVIII.

Liquid.	Experiment No.	T_{15}.	H.	μ_{15}.	A.
		dyne/cm.	cm.		cm.^{2}
Toluol	156	28·75	67·4	0·0064	0·00994
Water	129	74·12	69·6	0·0114	0·01042
9·5 per cent. alcohol	115	50·05	65·1	0·0175	0·01058
46·34 per cent. alcohol	89	28·93	59·2	0·0352	0·01131
81·02 per cent. alcohol	71	24·63	63·8	0·0227	0·01075
98·04 per cent. alcohol	59	22·78	63·5	0·0135	0·01054
Aniline	52	42·86	57·8	0·0550	0·01156

Table XVIII. contains a few results compiled from the above measurements, illustrating the relation between the cross-section of the jet (A) and the coefficient of viscosity (μ). The table is calculated for orifice III.; H is the effective head. The table shows that the cross-section increases with the viscosity.

TABLE of $\log \mu_n (x) + 10$.

$n = 2, 3, 4, 6. \qquad 0 \leqq x \leqq 1.$

x.	Log μ_2.	Log μ_3.	Log μ_4.	Log μ_6.
0·00	10·07248	9·47042	9·07248	8·52842
0·01	10·07247	9·47042	9·07248	8·52841
0·02	10·07241	9·47039	9·07247	8·52841
0·03	10·07232	9·47036	9·07245	8·52840
0·04	10·07219	9·47031	9·07242	8·52839
0·05	10·07203	9·47024	9·07238	8·52837
0·06	10·07183	9·47016	9·07234	8·52835
0·07	10·07160	9·47007	9·07229	8·52833
0·08	10·07133	9·46996	9·07223	8·52830
0·09	10·07102	9·46984	9·07216	8·52827
0·10	10·07068	9·46970	9·07209	8·52824
0·11	10·07030	9·46955	9·07200	8·52820
0·12	10·06988	9·46938	9·07191	8·52816
0·13	10·06943	9·46920	9·07181	8·52812
0·14	10·06895	9·46901	9·07170	8·52807
0·15	10·06843	9·46880	9·07159	8·52802
0·16	10·06787	9·46857	9·07147	8·52797
0·17	10·06728	9·46834	9·07133	8·52791
0·18	10·06665	9·46808	9·07119	8·52785
0·19	10·06599	9·46782	9·07105	8·52778
0·20	10·06529	9·46753	9·07089	8·52771
0·21	10·06455	9·46724	9·07073	8·52764
0·22	10·06379	9·46693	9·07056	8·52757
0·23	10·06298	9·46661	9·07038	8·52749
0·24	10·06215	9·46627	9·07019	8·52740
0·25	10·06128	9·46592	9·07000	8·52732
0·26	10·06037	9·46555	9·06980	8·52723
0·27	10·05943	9·46517	9·06959	8·52714
0·28	10·05846	9·46477	9·06937	8·52704
0·29	10·05745	9·46436	9·06914	8 52694

TABLE of log $\mu_n(x)+10$ (continued).

x.	Log μ_2.	Log μ_3.	Log μ_4.	Log μ_6.
0·30	10·05641	9·46394	9·06891	8·52684
0·31	10·05533	9·46350	9·06867	8·52673
0·32	10·05422	9·46305	9·06842	8·52662
0·33	10·05308	9·46259	9·06816	8·52650
0·34	10·05191	9·46211	9·06790	8·52639
0·35	10·05070	9·46162	9·06762	8·52627
0·36	10·04946	9·46111	9·06734	8·52614
0·37	10·04819	9·46059	9·06706	8·52601
0·38	10·04689	9·46005	9·06676	8·52588
0·39	10·04555	9·45951	9·06646	8·52575
0·40	10·04418	9·45894	9·06614	8·52561
0·41	10·04279	9·45837	9·06582	8·52547
0·42	10·04136	9·45778	9·06550	8·52532
0·43	10·03989	9·45717	9·06516	8·52517
0·44	10·03840	9·45656	9·06482	8·52502
0·45	10·03688	9·45593	9·06447	8·52487
0·46	10·03533	9·45528	9·06411	8·52471
0·47	10·03374	9·45463	9·06375	8·52454
0·48	10·03213	9·45396	9·06337	8·52438
0·49	10·03049	9·45327	9·06299	8·52421
0·50	10·02881	9·45257	9·06260	8·52404
0·51	10·02711	9·45186	9·06221	8·52386
0·52	10·02538	9·45114	9·06181	8·52368
0·53	10·02362	9·45040	9·06139	8·52350
0·54	10·02183	9·44965	9·06098	8·52331
0·55	10·02002	9·44889	9·06055	8·52312
0·56	10·01817	9·44811	9·06011	8·52293
0·57	10·01630	9·44732	9·05967	8·52273
0·58	10·01440	9·44652	9·05922	8·52253
0·59	10·01248	9·44570	9·05877	8·52232

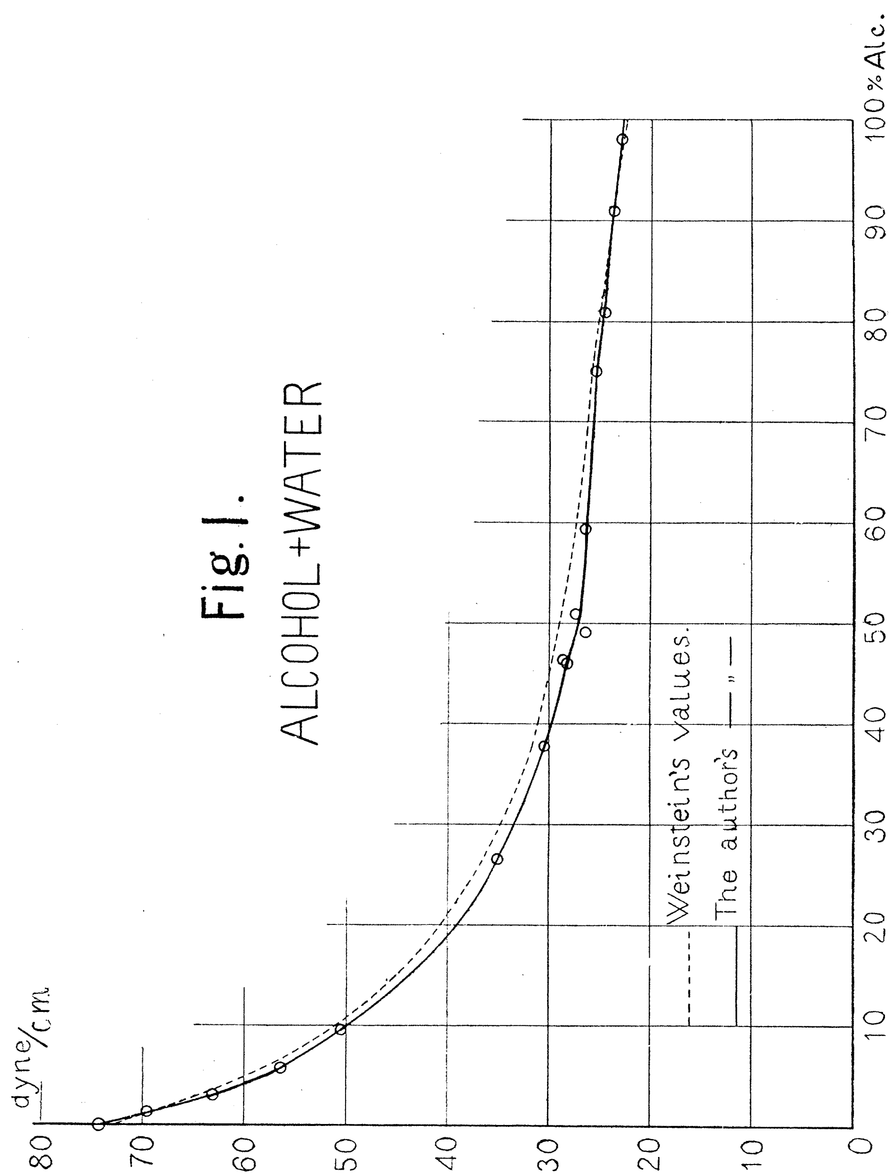
TABLE of $\log \mu_n(x)+10$ (continued).

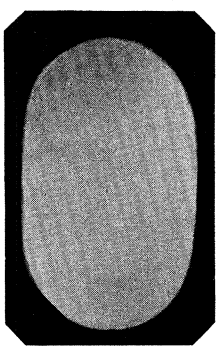
x.	Log μ_2.	Log μ_3.	Log μ_4.	Log μ_6.
0·60	10·01052	9·44487	9·05830	8·52212
0·61	10·00854	9·44403	9·05783	8·52191
0·62	10·00654	9·44317	9·05735	8·52169
0·63	10·00450	9·44231	9·05687	8·52147
0·64	10·00245	9·44143	9·05637	8·52125
0·65	10·00036	9·44053	9·05587	8·52103
0·66	9·99825	9·43963	9·05536	8·52080
0·67	9·99612	9·43871	9·05485	8·52057
0·68	9·99396	9·43778	9·05433	8·52034
0·69	9·99178	9·43684	9·05379	8·52010
0·70	9·98957	9·43588	9·05326	8·51986
0·71	9·98734	9·43491	9·05271	8·51961
0·72	9·98508	9·43393	9·05216	8·51936
0·73	9·98280	9·43294	9·05160	8·51911
0·74	9·98050	9·43194	9·05103	8·51886
0·75	9·97818	9·43092	9·05046	8·51860
0·76	9·97583	9·42989	9·04988	8·51834
0·77	9·97346	9·42885	9·04929	8·51807
0·78	9·97107	9·42780	9·04869	8·51781
0·79	9·96866	9·42674	9·04809	8·51753
0·80	9·96622	9·42566	9·04748	8·51726
0·81	9·96377	9·42457	9·04686	8·51698
0·82	9·96129	9·42347	9·04624	8·51670
0·83	9·95880	9·42236	9·04561	8·51641
0·84	9·95628	9·42124	9·04497	8·51613
0·85	9·95374	9·42010	9·04432	8·51583
0·86	9·95118	9·41896	9·04367	8·51554
0·87	9·94861	9·41780	9·04301	8·51524
0·88	9·94601	9·41663	9·04235	8·51494
0·89	9·94340	9·41545	9·04167	8·51463

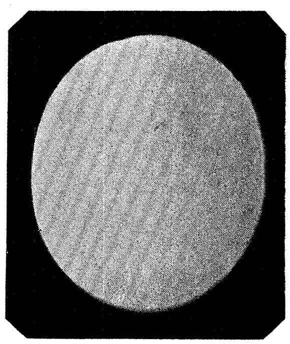
TABLE of log $\mu_n(x)+10$ (continued).

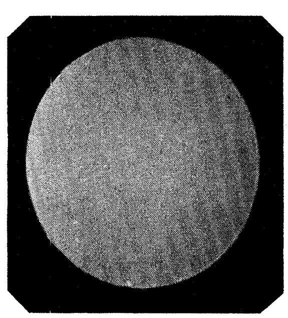
x.	Log μ_2.	Log μ_3.	Log μ_4.	Log μ_6.
0·90	9·94076	9·41426	9·04099	8·51433
0·91	9·93811	9·41306	9·04031	8·51402
0·92	9·93544	9·41185	9·03961	8·51370
0·93	9·93275	9·41062	9·03891	8·51338
0·94	9·93005	9·40939	9·03821	8·51306
0·95	9·92732	9·40814	9·03749	8·51274
0·96	9·92458	9·40689	9·03677	8·51241
0·97	9·92183	9·40562	9·03604	8·51208
0·98	9·91905	9·40434	9·03531	8·51174
0·99	9·91626	9·40305	9·03457	8·51141
1·00	9·91346	9·40175	9·03382	8·51106

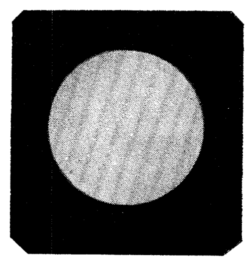
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

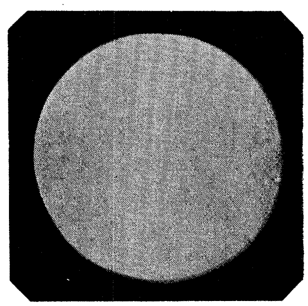


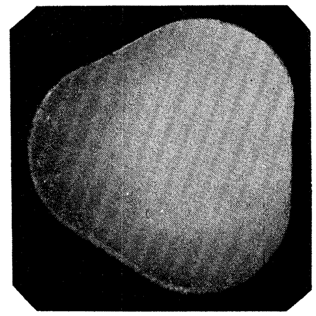

I. 1·671 mm.

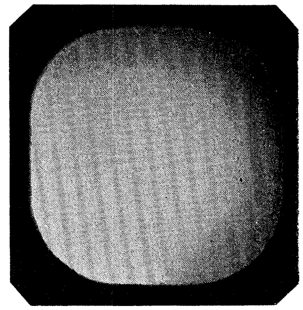

II. 1·545 mm.


III. 1·424 mm.


IV. 0·830 mm.


V. 1·386 mm.

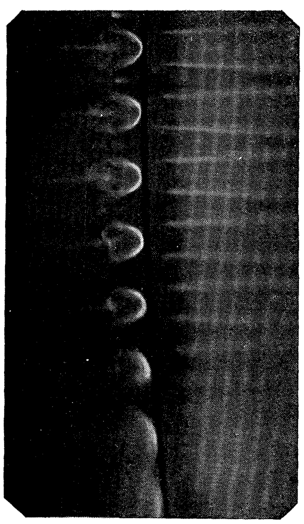

A. 2·202 mm.


B. 2·372 mm.

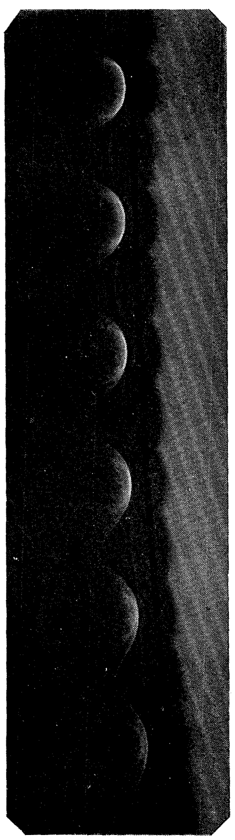
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

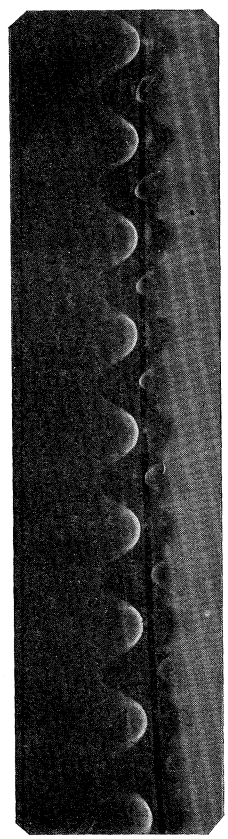
10 cm.



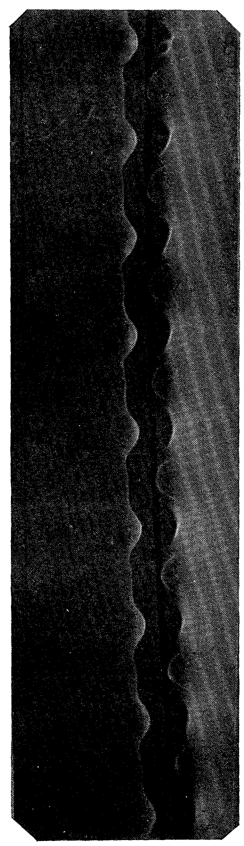
I. Fig. 1. Water.



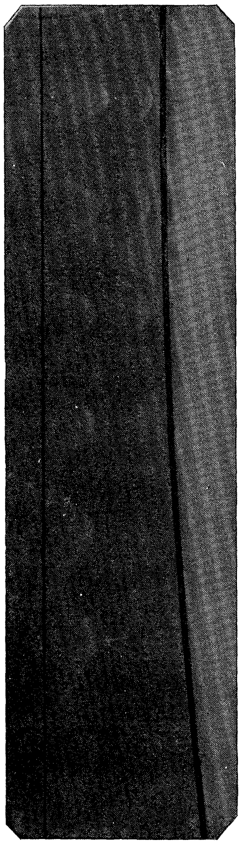
II. Fig. 2. 46·34 per cent. alcohol + 53·66 per cent. water.



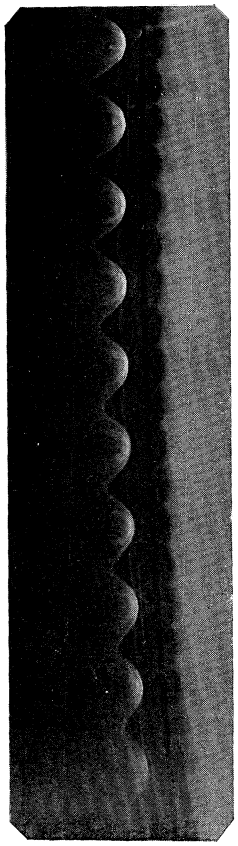
II. Fig. 3. Water.



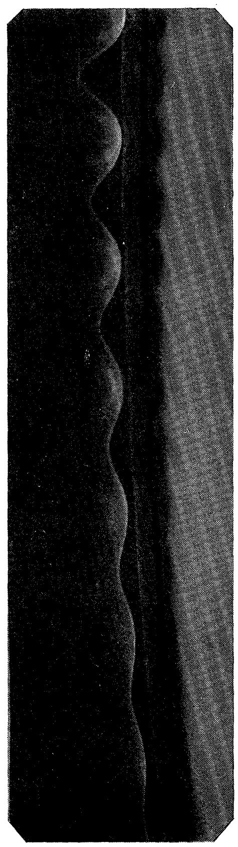
II. Fig. 4. Water.



III. Fig. 5. Aniline.



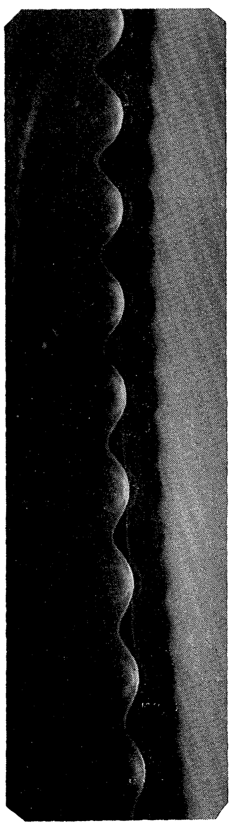
III. Fig. 6. Water.



III. Fig. 7. 46·11 per cent. alcohol + 53·89 per cent. water.

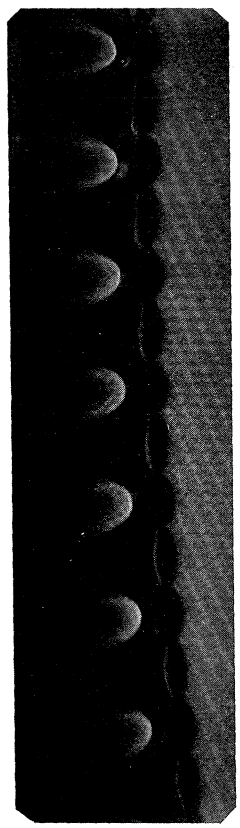
III. Fig. 8. 98·04 per cent. alcohol + 1·96 per cent. water.

10 cm.

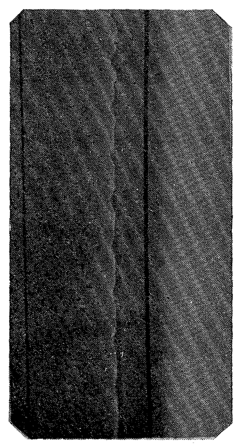


III. Fig. 1. 9·5 per cent. alcohol + 90·5 per cent. water.

III. Fig. 2. H_2SO_4 + Aq. $\rho_{15} = 1 \cdot 0813$.

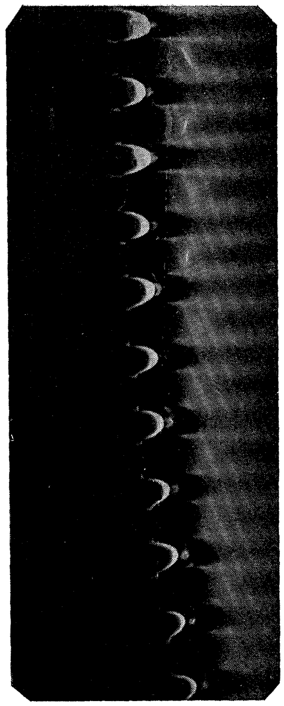


III. Fig. 3. Toluol.

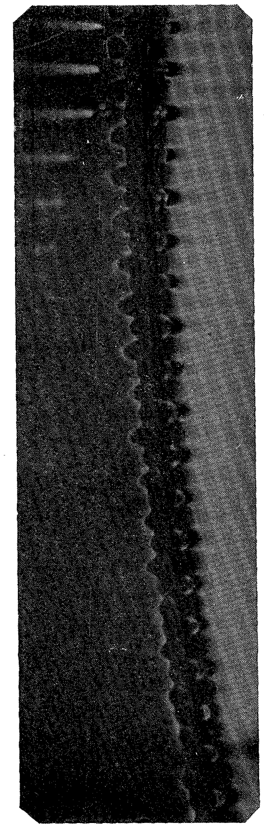


IV. Fig. 4. Water.

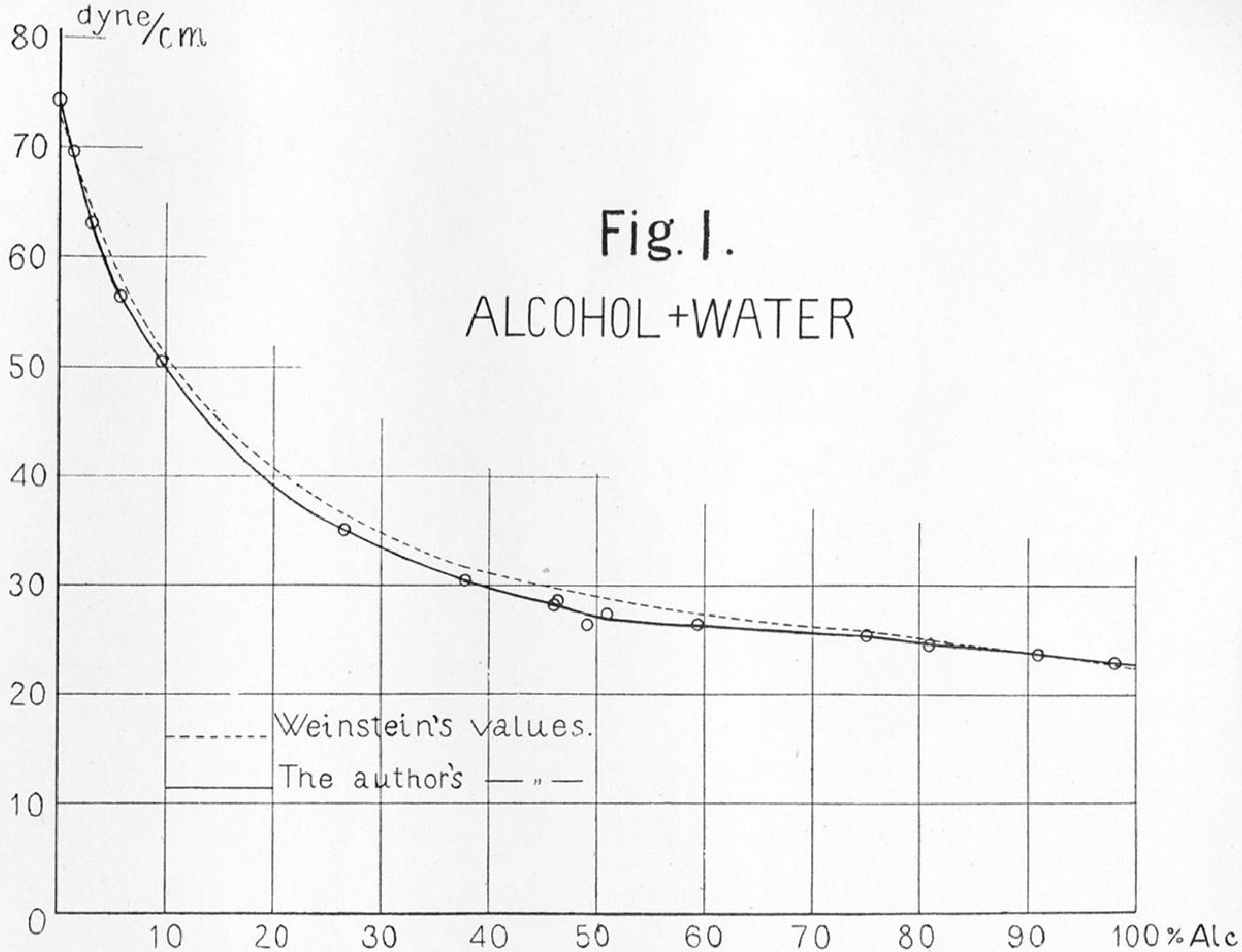
V. Fig. 5. Water.

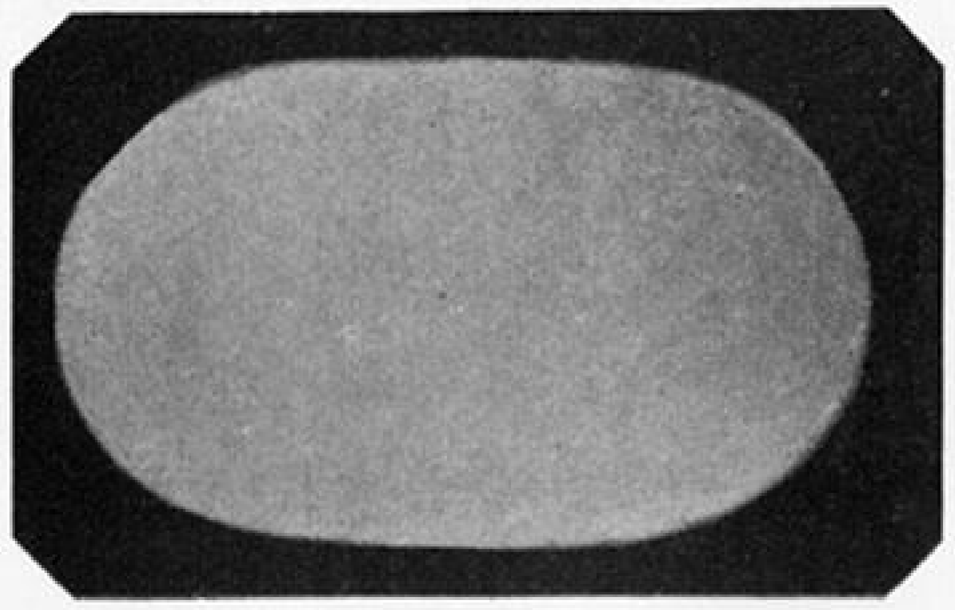


A. Fig. 6. Water.

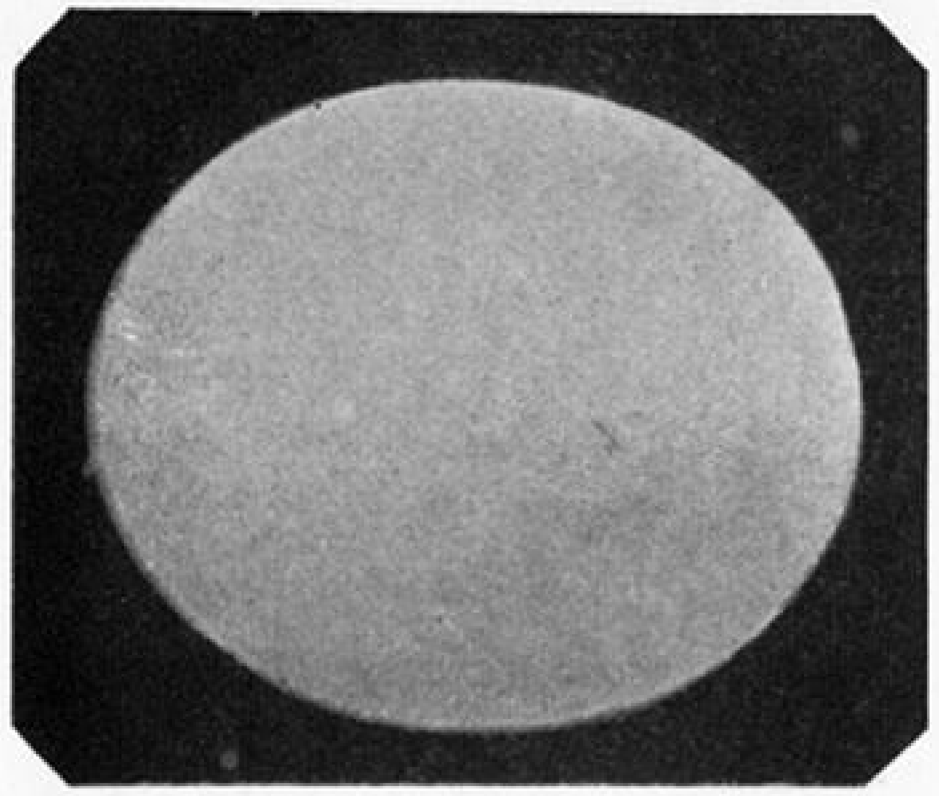


B. Fig. 7. Water.

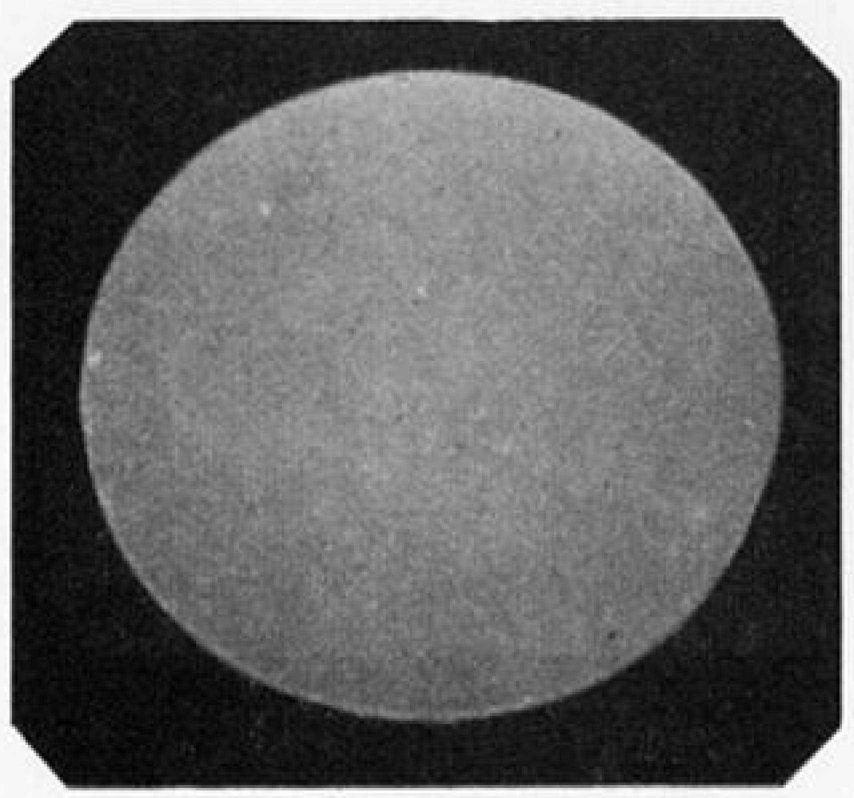




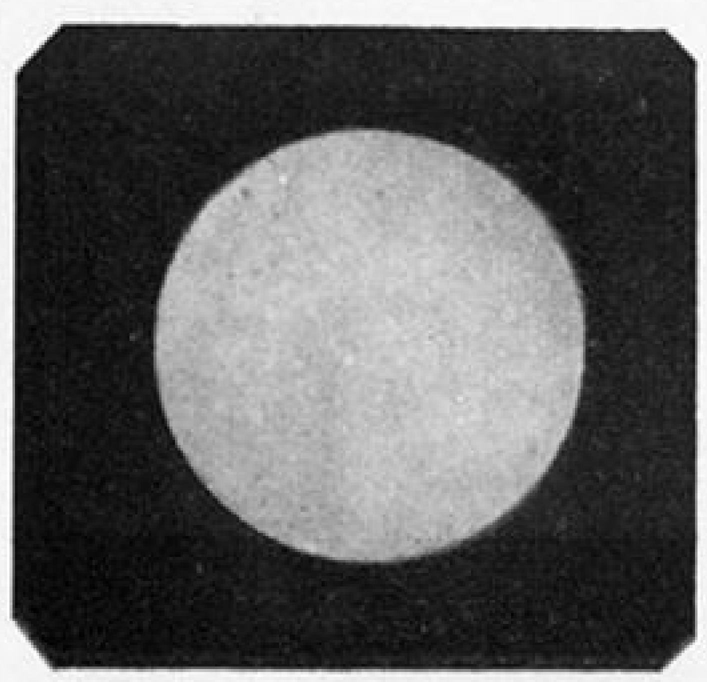
I. 1·671 mm.



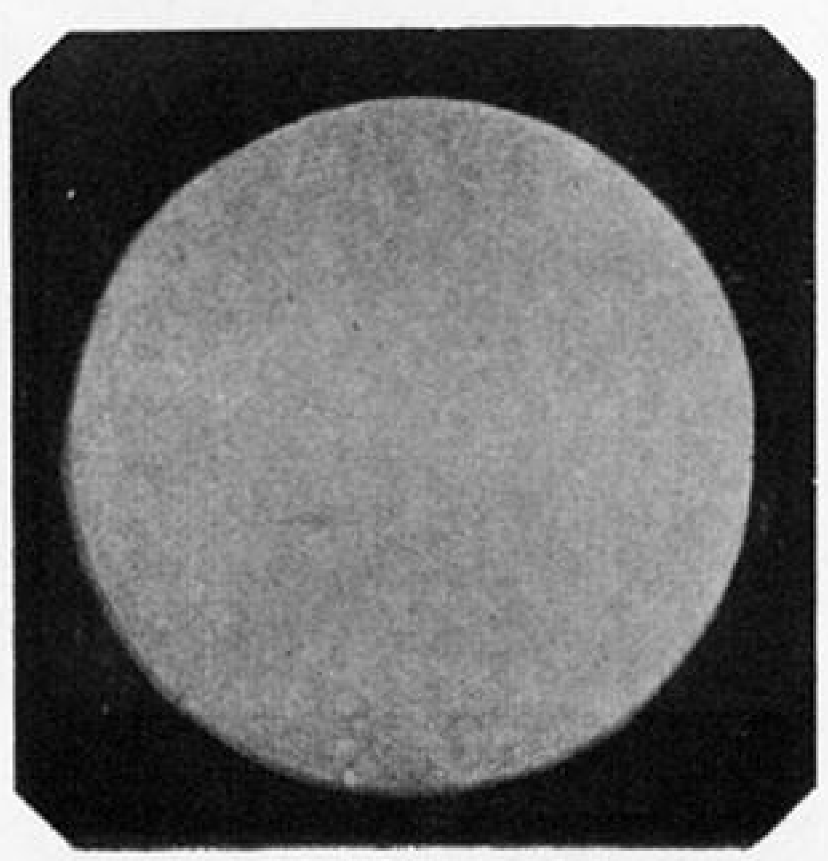
II. 1·545 mm.



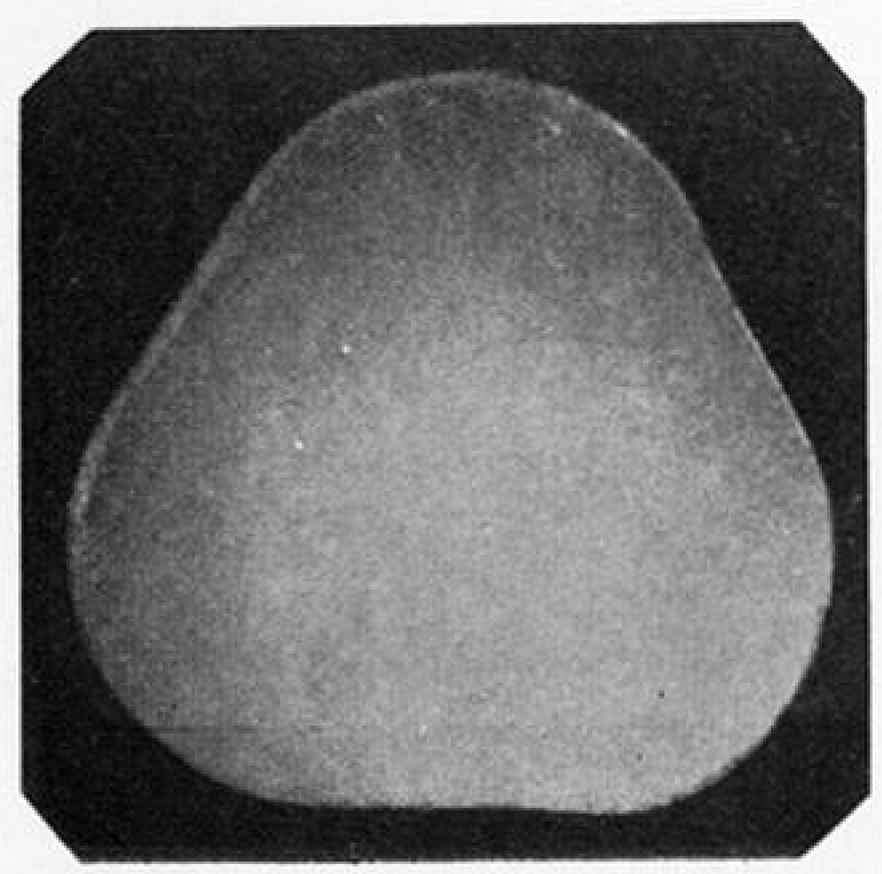
III. 1·424 mm.



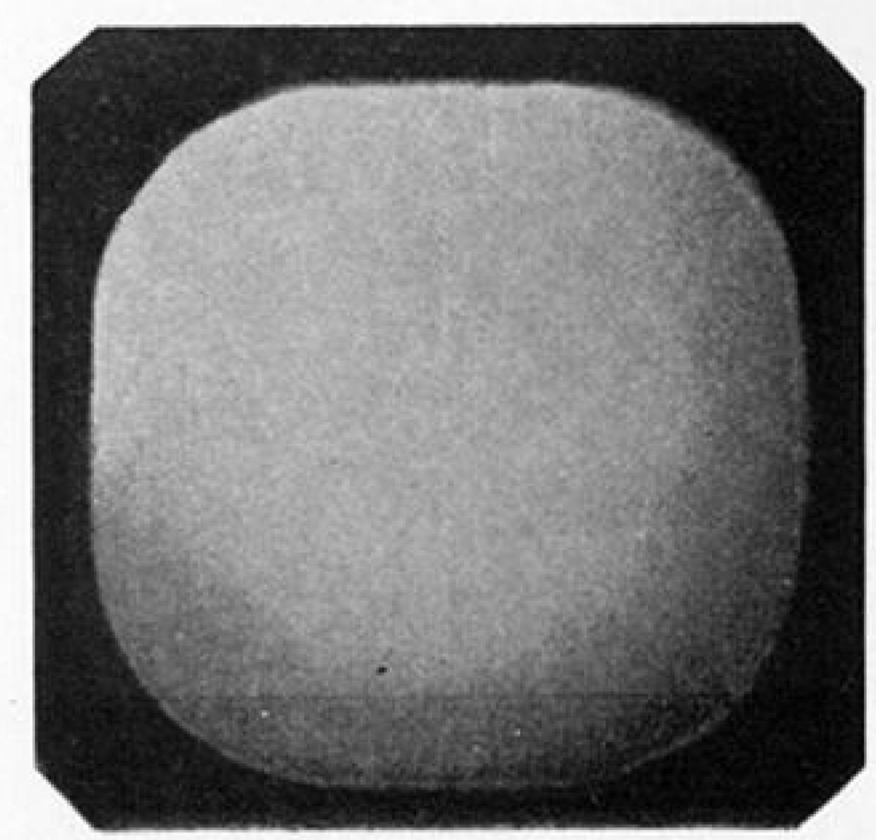
IV. 0·830 mm.



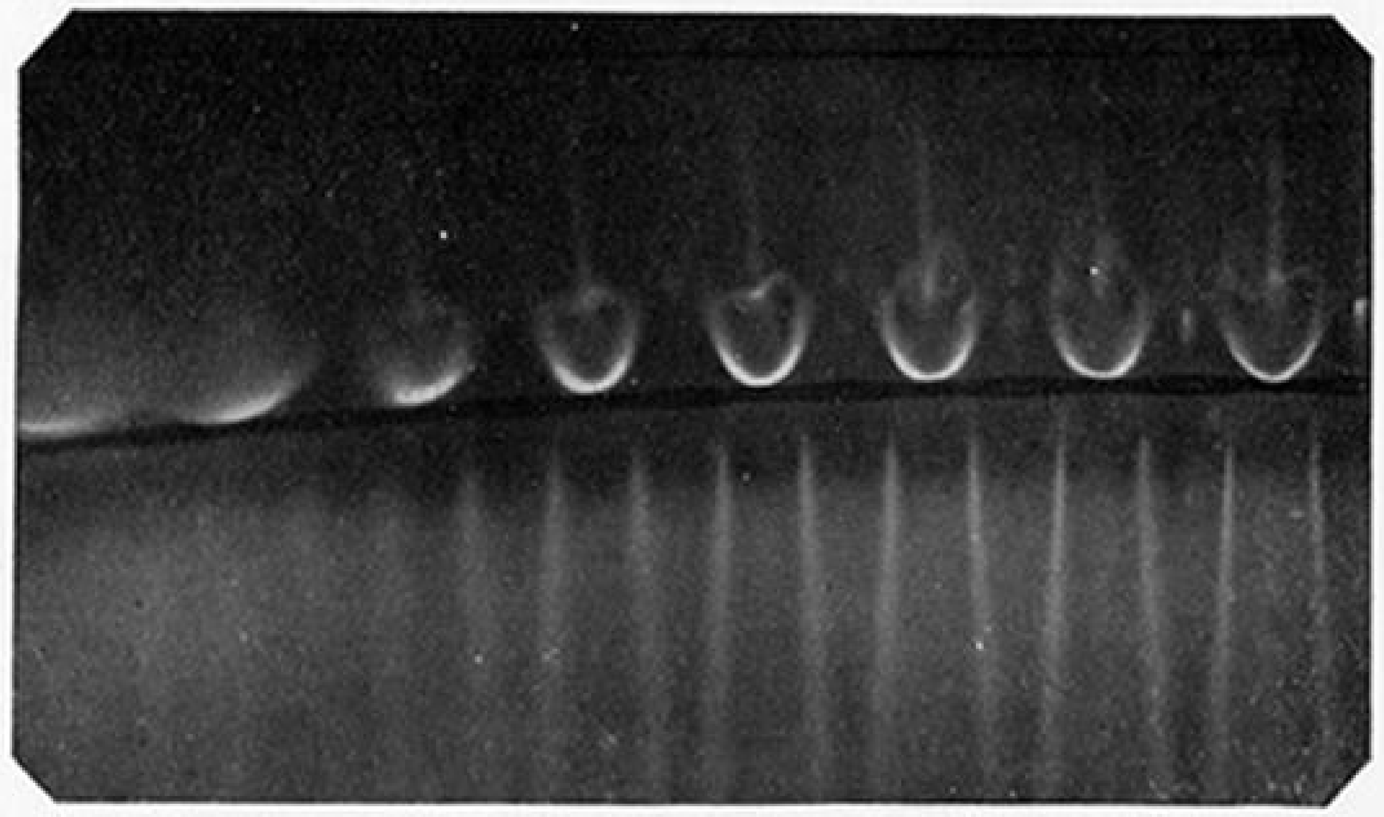
V. 1·386 mm.



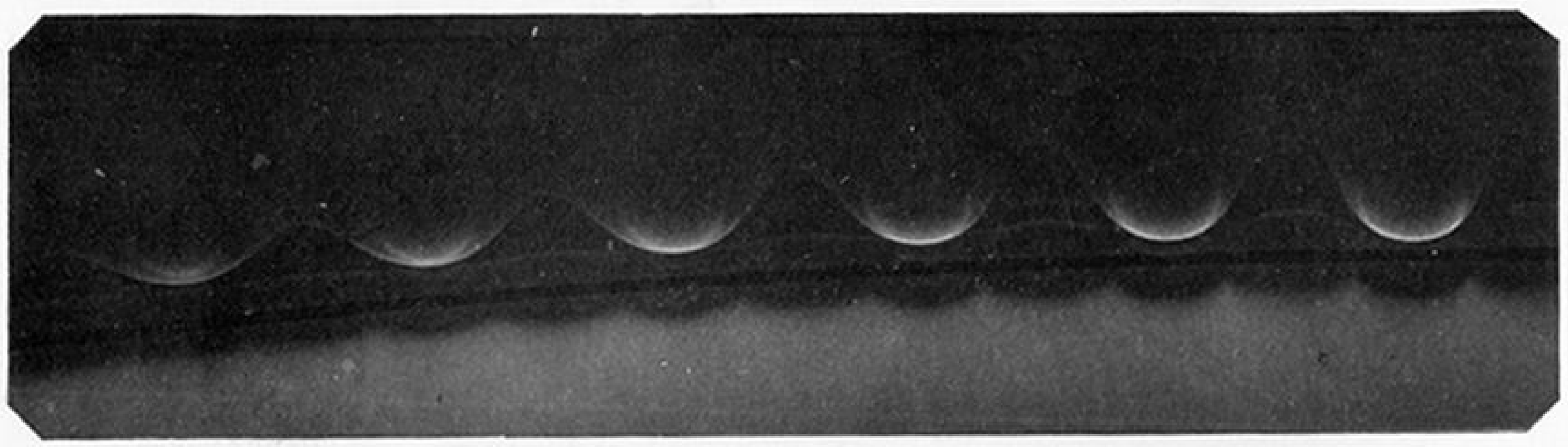
A. 2·202 mm.



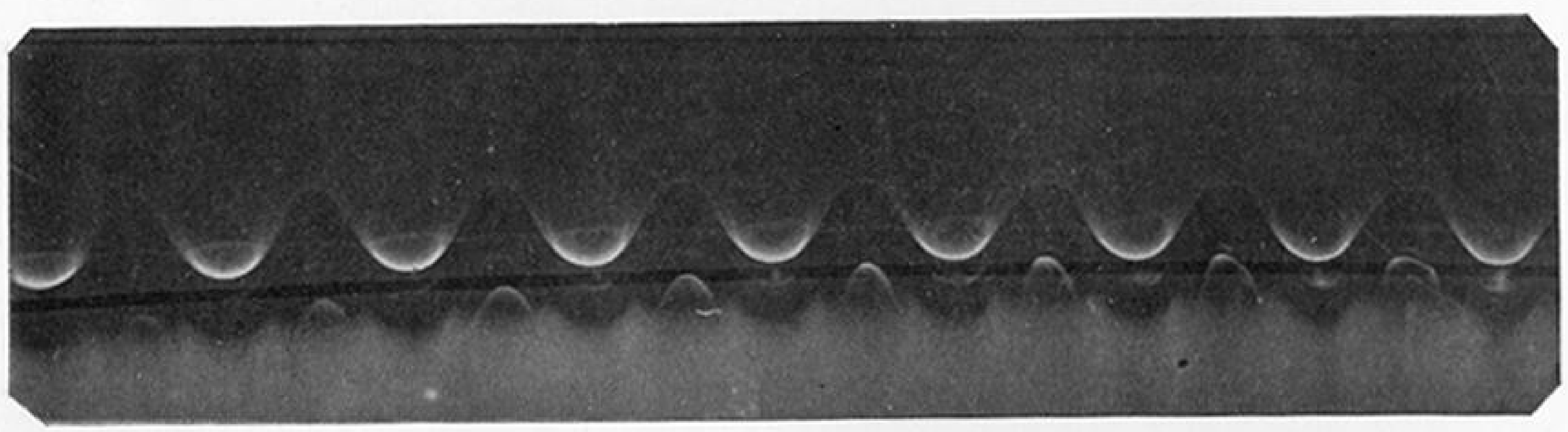
B. 2·372 mm.



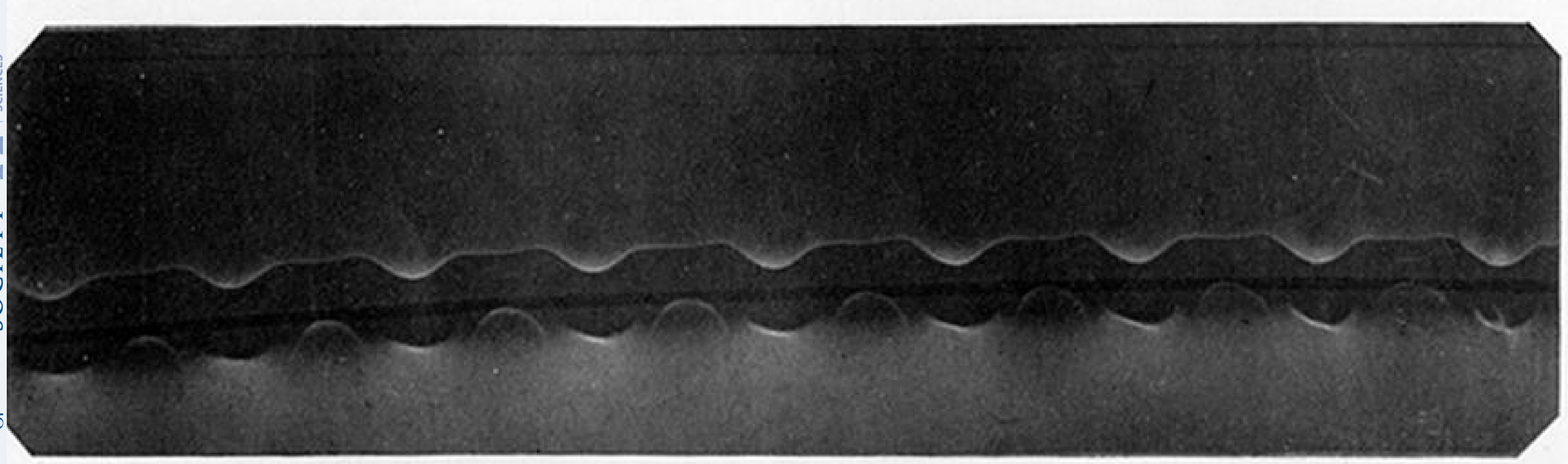
I. Fig. 1. Water.



II. Fig. 2. 46·34 per cent. alcohol + 53·66 per cent. water.

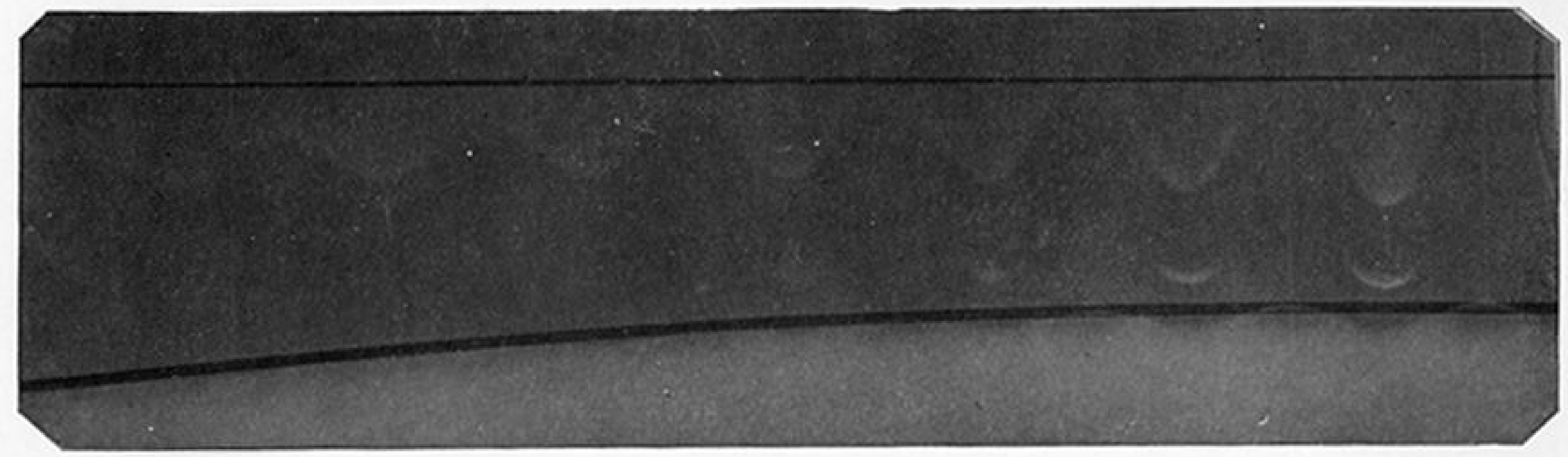


II. Fig. 3. Water.

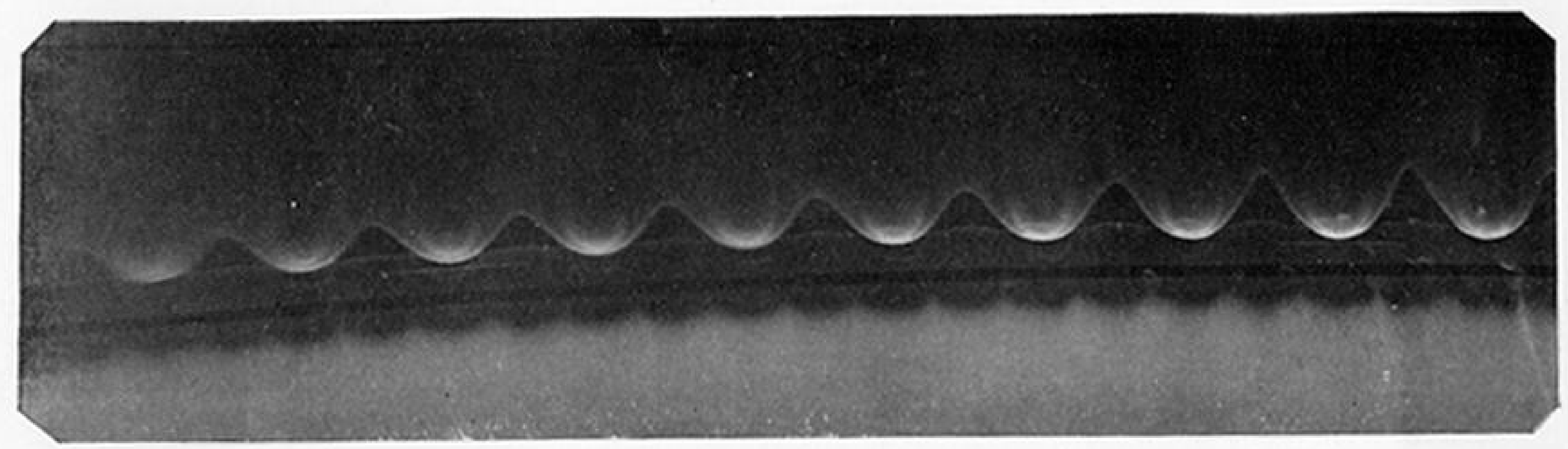


II. Fig. 4. Water.

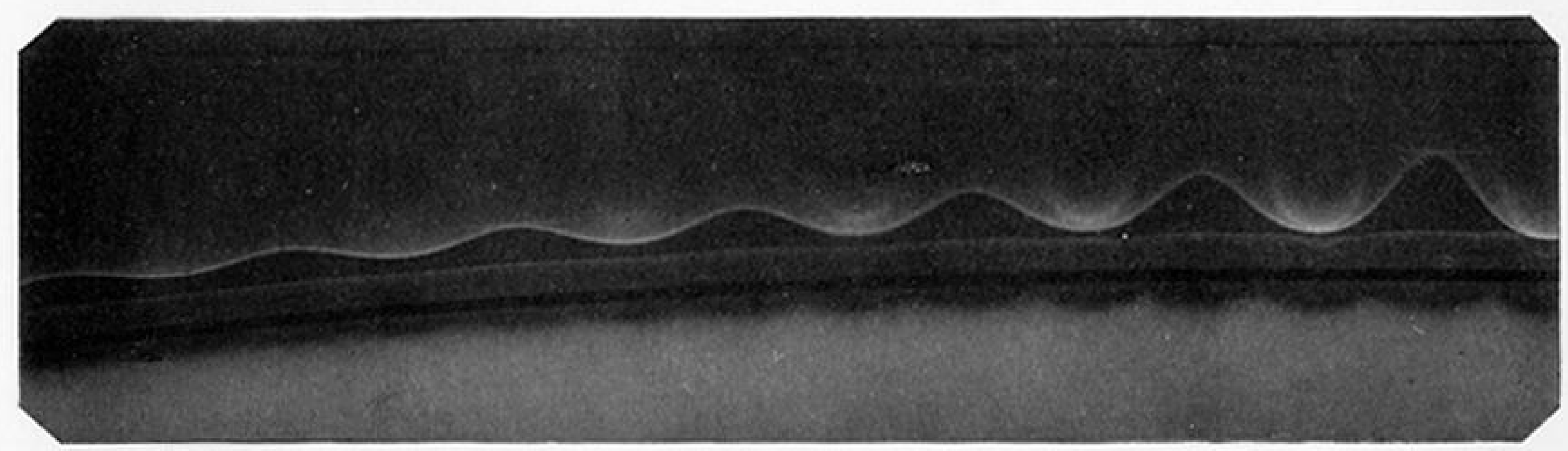
10 cm.



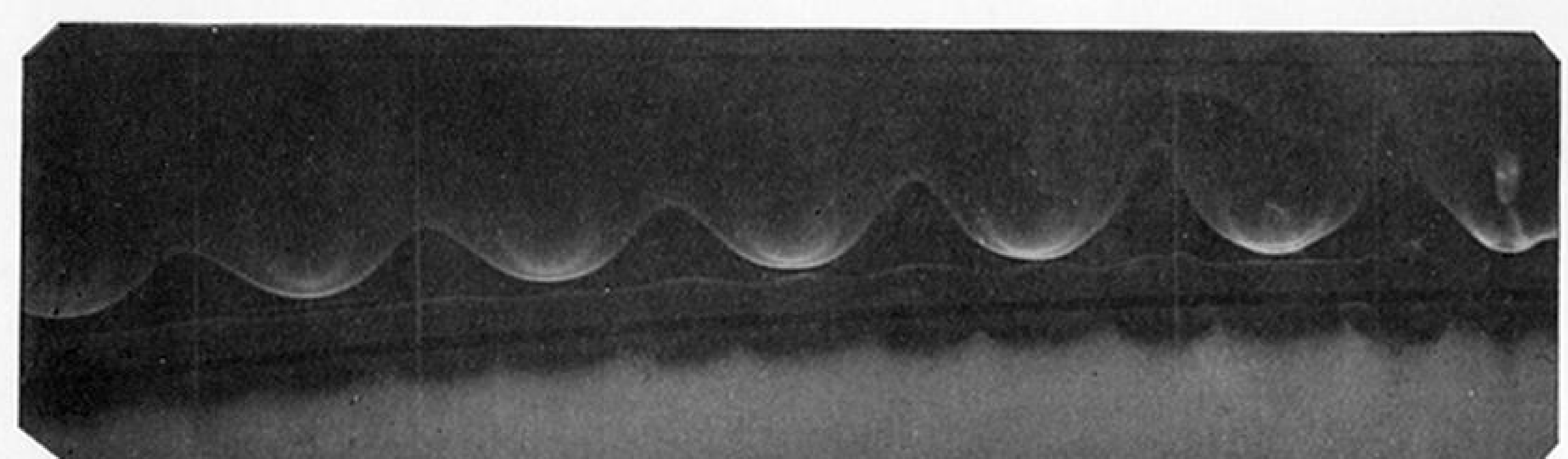
III. Fig. 5. Aniline.



III. Fig. 6. Water.

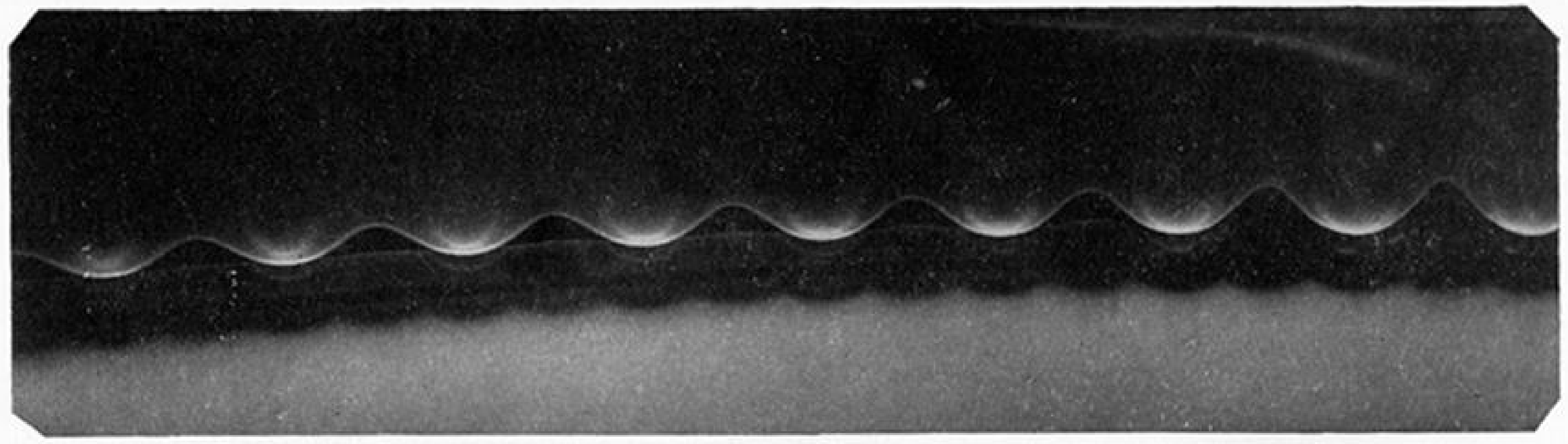


III. Fig. 7. 46·11 per cent. alcohol + 53·89 per cent. water.

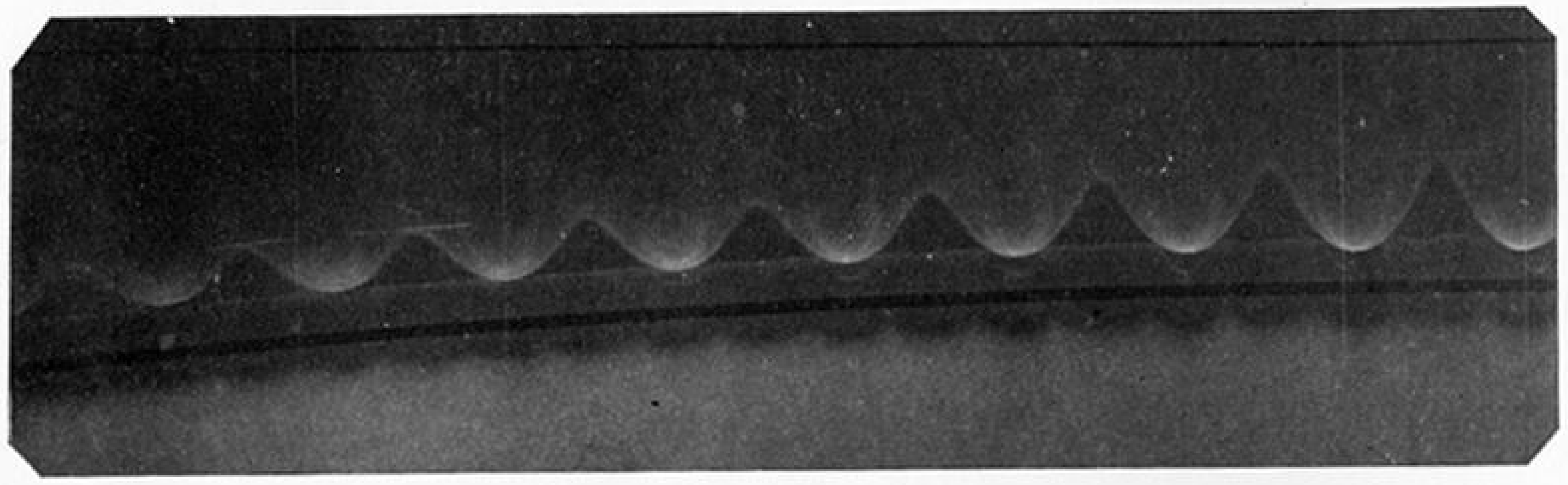


III. Fig. 8. 98·04 per cent. alcohol + 1·96 per cent. water.

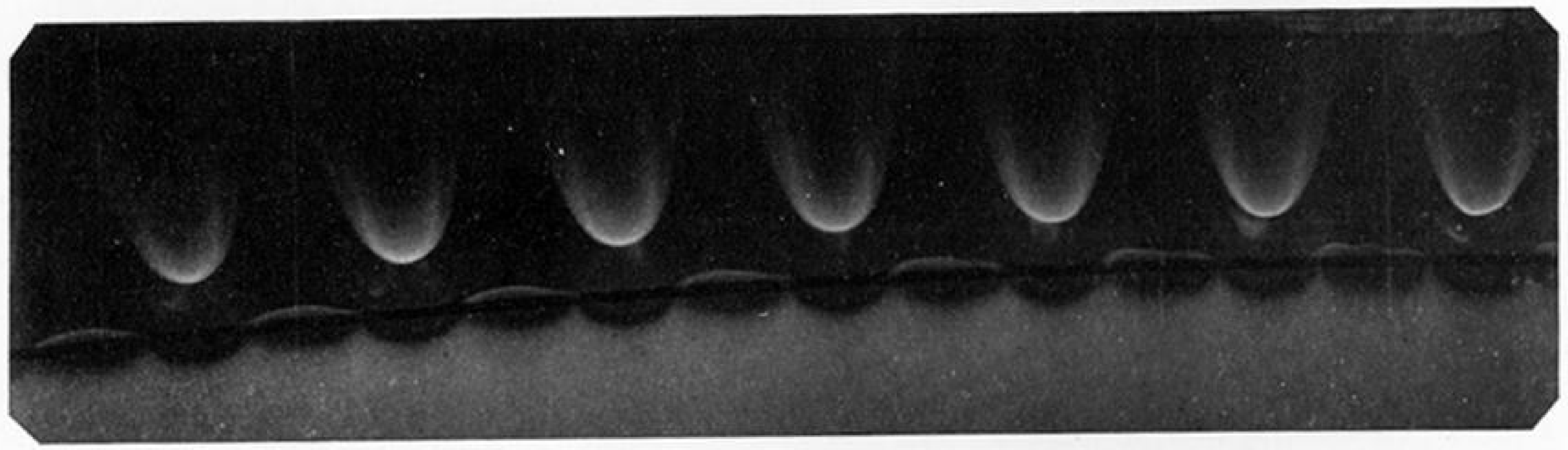
10 cm.



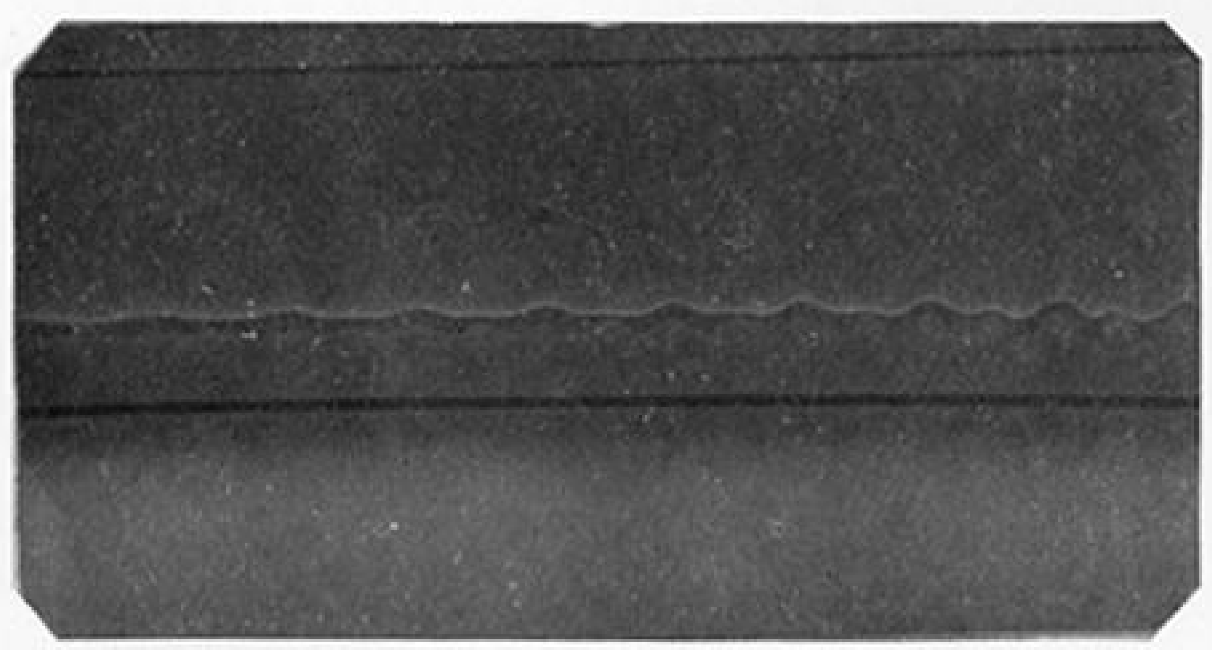
III. Fig. 1. 9·5 per cent. alcohol + 90·5 per cent. water.



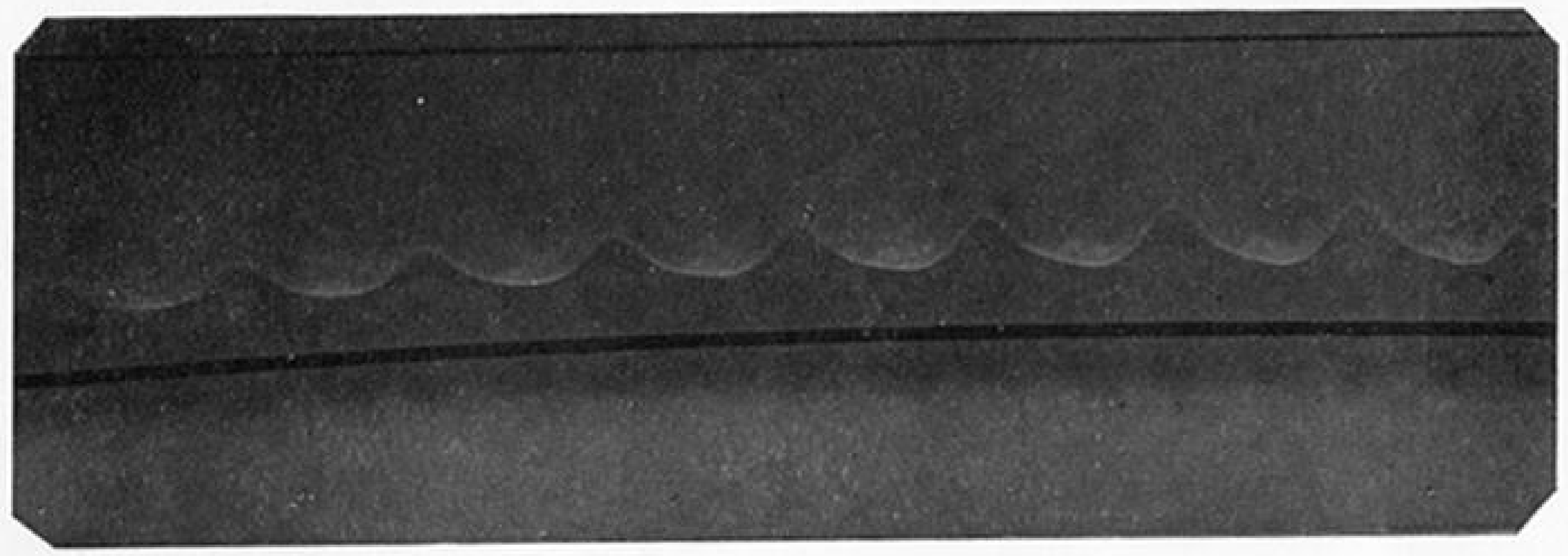
III. Fig. 2. $H_2SO_4 + Aq.$ $\rho_{15} = 1 \cdot 0813$.



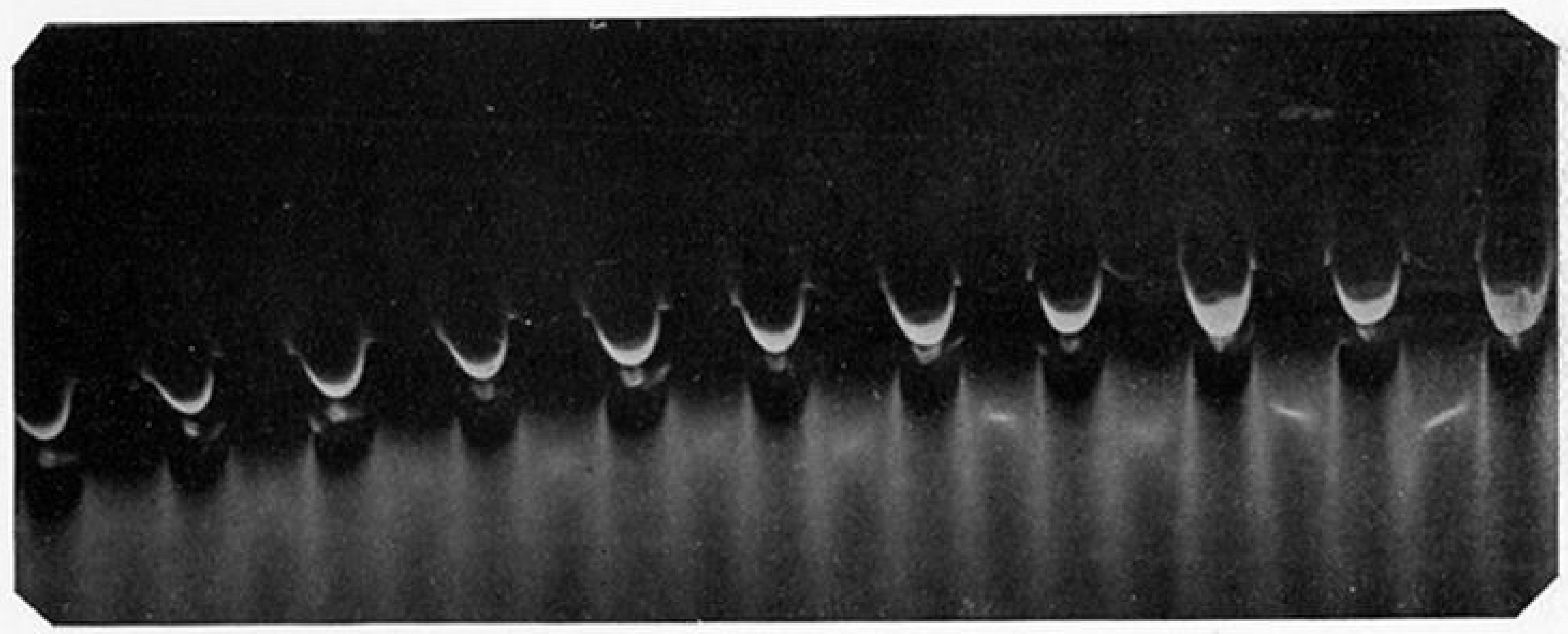
III. Fig. 3. Toluol.



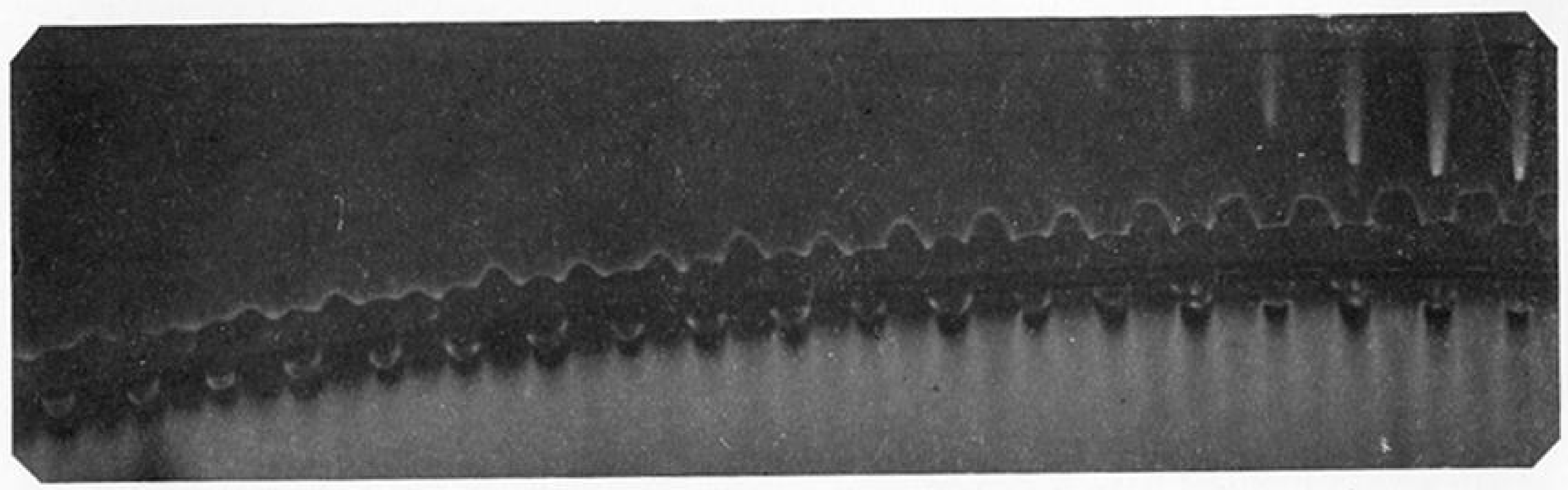
IV. Fig. 4. Water.



V. Fig. 5. Water.



A. Fig. 6. Water.



B. Fig. 7. Water.